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A METHOD FOR ESTIMATION OF MAXIMUM STRESSES AROUND A SMALL RECTANGULAR CUT-OUT IN A SHEET-STRINGER PANEL IN SHEAR

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A METHOD FOR ESTIMATION OF MAXIMUM STRESSES AROUND A SMALL RECTANGULAR CUT-OUT IN A SHEET-STRINGER PANEL IN SHEAR

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SUMMARY

A method for the estimation of the maximum stresses around a small rectangular cut-out in a sheet-stringer panel loaded in pure shear is presented; this method is based on a simplified application of the shear-lag theory. Comparisons indicate that the experimental maximum stringer and shear stresses agree reasonably well with the predictions, provided that neither the length nor the width of the cut-out is greater than the half-width of the panel.

INTRODUCTION

The problem of analyzing the stresses around a cut-out in a sheet-stringer panel under shear loads is difficult but can be simplified if the analysis is restricted to small cut-outs. Additional simplifications are permissible if it is agreed that, for most practical purposes, an estimate of the maximum stresses in the vicinity of the cut-out is sufficient and that a knowledge of the details of the stress distribution in other regions is not essential. The method of analysis presented herein takes full advantage of all possible simplifying assumptions in order that the maximum simplicity of application compatible with a reasonable degree of accuracy can be achieved. The principles used are similar to those used in reference 1 for the analysis of cut-outs in stiffened shells under bending loads. Comparisons are made of the stringer and shear stresses computed by this method with the experimental stresses obtained from strain surveys around a series of cut-outs.
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SYMBOLS

A enclosed area of cross section of box, square inches

A_R effective area of rib, square inches

A_S effective area of stringer, square inches

E Young's modulus of elasticity, ksi

G shear modulus of elasticity, ksi

K_R shear-lag parameter for stresses in rib-sheet system

K_S shear-lag parameter for stresses in stringer-sheet system

L half-length of cut-out, inches

T applied torque, kip-inches

a chordwise distance from edge of cut-out to edge of panel, inches

b half-width of cut-out, inches

t sheet thickness, inches

x spanwise distance from rib bounding cut-out, inches

y chordwise distance from stringer bounding cut-out, inches

w half-width of test panels (16.5 in.)

σ_R rib stress, ksi

σ_S stringer stress, ksi

τ_o basic shear stress in cover, ksi

τ_R shear stress in rib-sheet system caused by liquidating forces, ksi

τ_S shear stress in stringer-sheet system caused by liquidating forces, ksi

τ final shear stress in panel, ksi

Subscript:

max maximum

THEORETICAL ANALYSIS

Basic Assumptions and General Principles of Analysis

The structure considered for the analysis is a sheet-stringer panel with a rectangular cut-out bounded on two sides by stringers and on the other two sides by ribs that lie at right angles to the stringers. The stringers and ribs are assumed to be of constant section and the sheet is assumed to be of constant thickness. The dimensions of the cut-out are assumed to be small compared with the width of the panel. Under the simplifying assumptions on which the analysis is based, the cut-out need not be centrally located in the panel but it must not be close to an edge of the panel if the formulas are to be valid. The simplifying assumptions also imply that the results may be applied to a panel with a moderate amount of curvature.

The actual loading case investigated (fig. 1(a)) may be resolved into case I (fig. 1(b)) and case II (fig. 1(c)). In case I the basic shear stress τ_o is assumed to act along the outer edges of the panel and, also, at the edges of the cut-out. For such a loading, the shear stresses are everywhere equal to the basic shear stress τ_o, and there are no stringer stresses. In case II shear stresses, which are assumed to act only along the edges of the cut-out, are equal in magnitude but opposite in direction to those acting at the cut-out in case I. The shear stresses of case II will be called liquidating stresses because, when superposed on the stresses of case I, they neutralize the stresses at the edge of the cut-out and thus produce the condition of zero shear at the edge of the cut-out that exists in the actual case.

A qualitative theoretical study of case II, corroborated by a study of the experimental results, indicates that the stresses caused by the liquidating forces acting along the length of the panel are confined chiefly to the stringers bounding the cut-out and to the sheet lying between these stringers. Similarly, the stresses caused by the liquidating forces acting across the width of the panel are confined chiefly to the ribs bounding the cut-out and to the sheet lying between these ribs. In order to simplify the problem of analysis, the actual structure (fig. 2(a)) is replaced by a simplified structure consisting of the afore-mentioned parts of the actual structure which carry the major share of the stresses caused by the cut-out. This simplified structure, shown in figures 2(b) and 2(c), consists of four sheet panels, each bounded by two stringers or two ribs. It is assumed that each of these four panels may be analyzed by the formulas for the free panel given in reference 2 (appendix B, Case 2), or by simplifications of these formulas given in reference 1. A qualitative representation of the stresses expected under the assumptions outlined is given in figures 2(b) and 2(c).

Stresses in Stringer-Sheet System

<u>Stringer stresses</u>.- The uniformly distributed, liquidating shear stresses, applied to the edges of the cut-out parallel to the stringers, produce stresses which, because of the antisymmetry of the stringer-stress distribution in the vicinity of the cut-out, are zero at the center line of the cut-out and increase linearly to a maximum at the ribs (fig. 2(c)). The maximum stringer stress is

$$\sigma_{S_{max}} = \pm \frac{\tau_o tL}{A_S}$$

If equation (18a) of reference 1 is written in terms of stress, the stringer stress beyond the cut-out can be expressed as

$$\sigma_S = \sigma_{S_{max}} e^{-K_S x} \qquad (1)$$

The shear-lag parameter K_S is defined by the equation

$$K_S{}^2 = \frac{Gt}{EbA_S}$$

which is analogous to equation (18b) of reference 1. The stresses obtained from equation (1) are the final stringer stresses in the panel, for no stringer stresses existed before the liquidating stresses were applied.

Shear stresses.- The shear stresses caused by the liquidating forces are obtained from equation (18c) of reference 1, which, in the notation of the present report, is

$$\tau_S = \tau_o K_S L e^{-K_S x}$$

The final shear stresses in the sheet, obtained by adding the basic shear stress to the shear stresses caused by the liquidating forces, are

$$\tau = \tau_o\left(1 + K_S L e^{-K_S x}\right)$$

Stresses in Rib-Sheet System

Rib stresses.- The liquidating shear forces acting on the rib edges of the cut-out set up a system of stresses in the ribs and the sheet between them. By reasoning analogous to that used in the section "Stresses in Stringer-Sheet System," the stresses in the ribs and sheet may be determined. The liquidating forces applied to the ribs produce stresses in the ribs that are zero at the center line of the cut-out and increase linearly to maximums at the stringers. At the stringers the maximum stresses in the ribs are given by

$$\sigma_{R_{max}} = \pm \frac{\tau_o tb}{A_R}$$

The rib stress away from the cut-out is

$$\sigma_R = \sigma_{R_{max}} e^{-K_R y} \qquad (2)$$

in which the shear-lag parameter K_R is defined by

$$K_R^2 = \frac{Gt}{ELA_R}$$

Because there were no rib stresses before the liquidating forces were applied, equation (2) gives the final values for the rib stresses.

Shear stresses.- As is the case for the shear stresses in the stringer-sheet system, the shear stresses in the sheet between ribs decrease exponentially according to the equation

$$\tau_R = \tau_o K_R b e^{-K_R y}$$

The final shear stresses in the sheet are the sum of the stresses caused by the liquidating forces and the basic shear stress, or

$$\tau = \tau_o \left(1 + K_R b e^{-K_R y}\right) \tag{3}$$

The maximum value that the expression $K_R y$ may reach is $K_R a$, which, in many practical cases, has a numerical value appreciably less than unity. In such cases it is sufficiently accurate to assume that the shear stresses are uniformly distributed over the length a. Equation (3) can then be replaced by the simpler equation derived from the consideration of the static equilibrium of the shear stresses

$$\tau = \tau_o \left(1 + \frac{b}{a}\right) \tag{4}$$

It should be noted that, because the shear stresses given by equation (4) are constant, the rib stresses away from the cut-out statically consistent with these shear stresses would decrease linearly from a maximum at the cut-out to zero at the edges of the panel instead of exponentially as given by equation (2).

EXPERIMENTAL INVESTIGATION

Test Specimens and Procedure

A series of panels with different sizes of cut-out was tested in shear. The basic data for the test specimens are given in table I. The details of a typical test panel attached as a cover to a torsion box are shown in figure 3. All panels were 24S-T aluminum-alloy sheet 0.032 inch thick reinforced with 24S-T aluminum-alloy extruded-angle stiffeners, 3/4 by 3/8 by 3/32 inch, spaced 3 inches. The ribs bounding the cut-out were 24S-T aluminum-alloy extruded angles 1 by 1 by 1/8 inch.

The torsion box was loaded by means of the torque loading frame shown in figure 4. For each test run, the maximum applied load was smaller than the buckling load of the sheet.

The strain surveys were made on all specimens with 2-inch Tuckerman optical strain gages. The shear strains in the sheet were obtained from strains measured by gages placed at angles of 45° and 135° to the direction of the stringers. During each test run, gage readings were taken at zero load and at three equal increments of load to the maximum load. The load was released and checks of the zero readings were made. The strain readings were plotted against load and a straight line was drawn through the points. If the line did not intercept the origin of the plot as drawn, a parallel line was drawn to fulfill this condition; and a new value of strain obtained from the new line at maximum load was used for the computation of the corresponding stress. Such new lines were necessary in only a few cases.

Rib strains were not measured because the ribs were so large that the stresses in them were too small to be measured with sufficient accuracy. It was necessary to use large angles for the ribs because they were relied upon to prevent general instability failure of the stiffened panels.

Results and Discussion

For purposes of comparison of the observed stresses with the theory, the measured strains were converted to the corresponding stresses. The stringer stresses were computed with an assumed value of Young's modulus of 10,600 ksi. The shear strains were computed as the difference of the strain readings taken on the sheet at angles of 45° and 135° to the direction of the stringers and were converted to shear stresses with an assumed modulus of 4000 ksi. The basic shear stress was computed by the formula

$$\tau_o = \frac{T}{2At}$$

Stringer stresses.- Figures 5 to 15 show the spanwise distribution of the experimental and calculated stringer stresses; the method of designating stringers is shown in figure 2(a). The discontinuity in the calculated stringer stress at the edge of the cut-out is accounted for by the fact that the effective area A_S changed there. Beyond the cut-out, the area A_S included, in addition to the area of the stringer along the edge of the cut-out, the effective area of sheet on either side of the stringer; within the cut-out, however, the effective area of sheet was reduced by the area of sheet removed by the cut-out. The effective width of sheet on either side of the stringer was assumed to be $1\frac{1}{2}$ inches, which is one-half the actual stringer spacing.

A consideration of the observed and calculated values of stringer stresses indicates that, when the cut-out is small, the calculated maximum stresses are conservative. In figures 8 and 9, in which the length of the cut-out $2L$ is about equal to, or larger than, the half-width of the panel, the calculated stresses are very conservative; whereas in figures 12 and 15, in which the width of the cut-out $2b$ is larger than the half-width of the panel, the stresses are unconservative. It thus appears that, as the length of the cut-out increases, the calculated values of stringer stress become more conservative and, as the width of the cut-out increases, the stresses become less conservative and finally unconservative.

A study of figures 5 to 15 shows that the main stringers - that is, the stringers bounding the cut-outs - carried by far the highest stresses of all stringers. The stringers immediately adjacent had much lower stresses, which amounted at most to about one-fourth the stresses in the main stringers. In the remaining stringers, the stresses were low. These results may be used as a guide if it should be considered desirable to estimate the stresses in those stringers for which the simplified analysis yields no solution.

Shear stresses in stringer-sheet system.- The calculated shear stresses in the stringer-sheet system decreased from a maximum value at the edge of the cut-out to the basic shear stress at a large distance from the cut-out (fig. 2(b)); the observed stresses are shown in figures 16 to 27. A study of the observed stresses shows that they are in qualitative agreement with the calculated values for those specimens in which only one bay is interrupted by the cut-out (figs. 16 and 19 to 21). Quantitatively, the calculated stresses were slightly conservative for short cut-outs and became more conservative as the length of the cut-out increased.

When more than one bay was interrupted by the cut-out, the calculated stresses were either somewhat conservative or in fair agreement with the observed stresses in the bays adjacent to the main stringers as long as the width of the cut-out was less than the half-width of the box (figs. 17, 18, 22, 23, 25, and 26). When the width of the cut-out was increased beyond the half-width of the box, the calculated stresses became unconservative (figs. 24 and 27).

In interrupted bays not adjacent to the main stringers, the observed stresses reached a maximum value at some distance from the cut-out and dropped off as the cut-out was approached. (See fig. 23, bays 1 and 2.) No calculated curves are shown for such bays. A study of the figures indicates, however, that the maximum shear stress in such a bay is never higher than the maximum stress in the interrupted bay adjacent to the main stringer. A close or a conservative estimate of the maximum shear stress results, therefore, if the maximum stress calculated for the stringer-sheet system is assumed to exist in all the bays that are interrupted by the cut-out, provided that the width of the cut-out is less than the half-width of the box. If the width of the cut-out is more than the half-width of the box, the calculated stresses are unconservative.

Shear stresses in rib-sheet system.- A study of the experimental shear stresses in the sheet lying between the ribs (figs. 16 to 27) indicates that these stresses follow patterns which are easily discerned qualitatively but difficult to describe quantitatively. The calculated stresses shown are based on formula (4). A study of the results indicates that the maximum experimental stresses show some tendency to be higher than the calculated stresses and that the maximum may occur either in the bay adjacent to the cut-out or in the bay lying along the edge of the panel.

Shear stresses in corners of panel.- The determination of the shear stresses in that part of the sheet which lies outside of the main stringers as well as outside of the ribs is beyond the scope of the highly simplified method presented in this paper. Inspection of the experimental evidence indicates that the shear stresses in these regions are not likely to be materially higher than the basic shear stress except, perhaps, for very long cut-outs.

CONCLUSIONS

Comparisons are presented in this paper between calculated and experimental stresses around small rectangular cut-outs in sheet-stringer panels subjected to shear loads. It is concluded from these comparisons that the formulas presented give a reasonably accurate estimate of the maximum stresses, provided that neither the width nor the length of the cut-out is more than the half-width of the panel.

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REFERENCES

1. Kuhn, Paul, and Chiarito, Patrick T.: Shear Lag in Box Beams - Methods of Analysis and Experimental Investigations. NACA Rep. No. 739, 1942.

2. Kuhn, Paul: Stress Analysis of Beams with Shear Deformation of the Flanges. NACA Rep. No. 608, 1937.

TABLE I

BASIC DATA FOR SHEAR TESTS OF

PANELS WITH SMALL CUT-OUTS

Test	t (in.)	L (in.)	b (in.)	a (in.)	K_S	T (in.-kips)	τ_0 (ksi)
1	0.0325	2	$1\frac{1}{2}$	15	0.2055	135	3.83
2	.0325	2	$4\frac{1}{2}$	12	.1185	135	3.83
3	.0325	2	$7\frac{1}{2}$	9	.0918	90	2.55
4	.0352	7	$1\frac{1}{2}$	15	.2095	135	3.54
5	.0352	14	$1\frac{1}{2}$	15	.2095	108	2.83
6	.0352	21	$1\frac{1}{2}$	15	.2095	108	2.83
7	.0348	7	$4\frac{1}{2}$	12	.1208	117	3.11
8	.0348	7	$7\frac{1}{2}$	9	.0936	117	3.11
9	.0348	7	$10\frac{1}{2}$	6	.0791	90	2.39
10	.0318	13	$4\frac{1}{2}$	12	.1180	90	2.61
11	.0318	13	$7\frac{1}{2}$	9	.0915	63	1.83
12	.0318	13	$10\frac{1}{2}$	6	.0772	54	1.56

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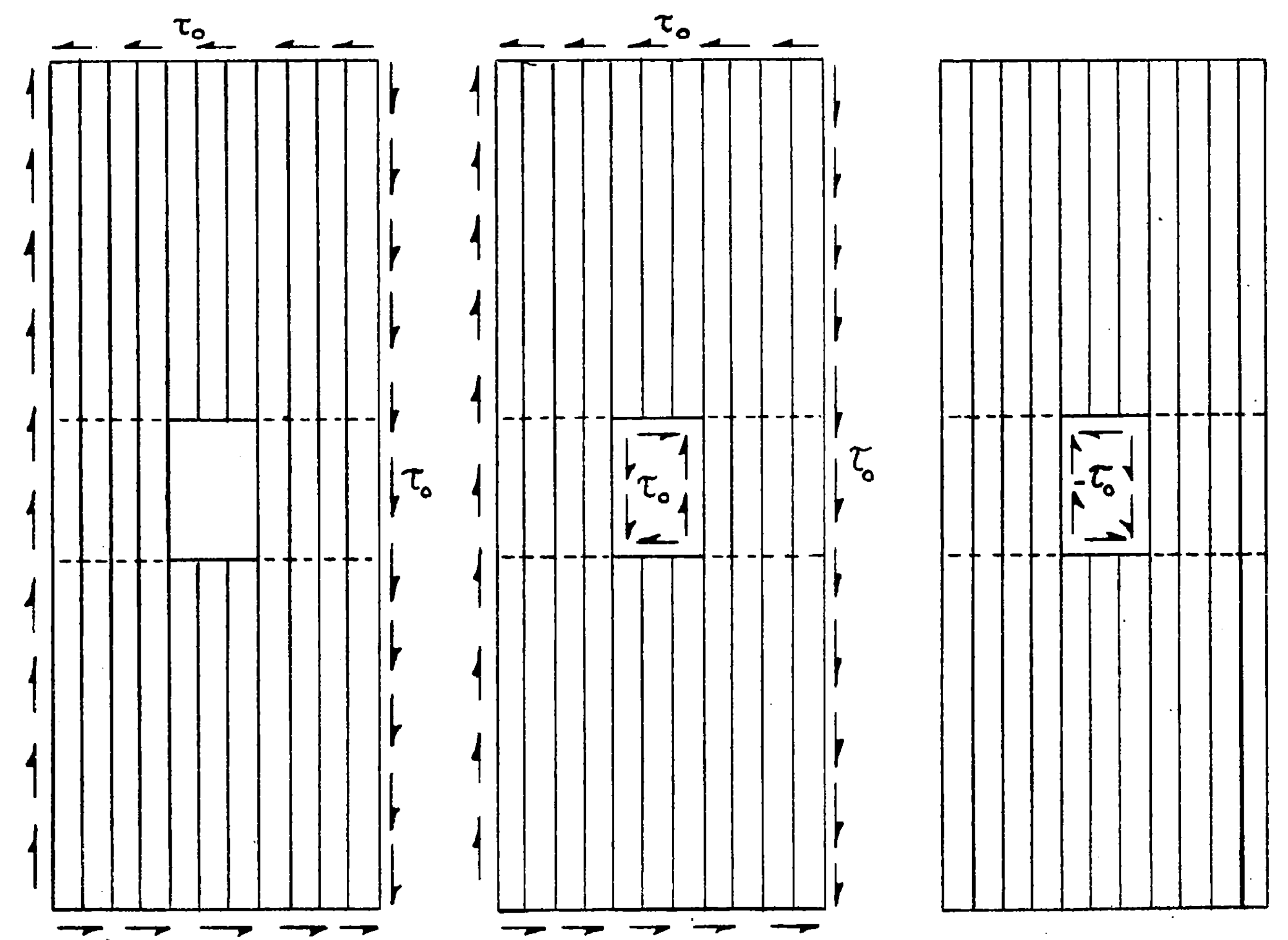

(a) Actual case. (b) Case I. (c) Case II.

Figure 1. - Panel loading cases.

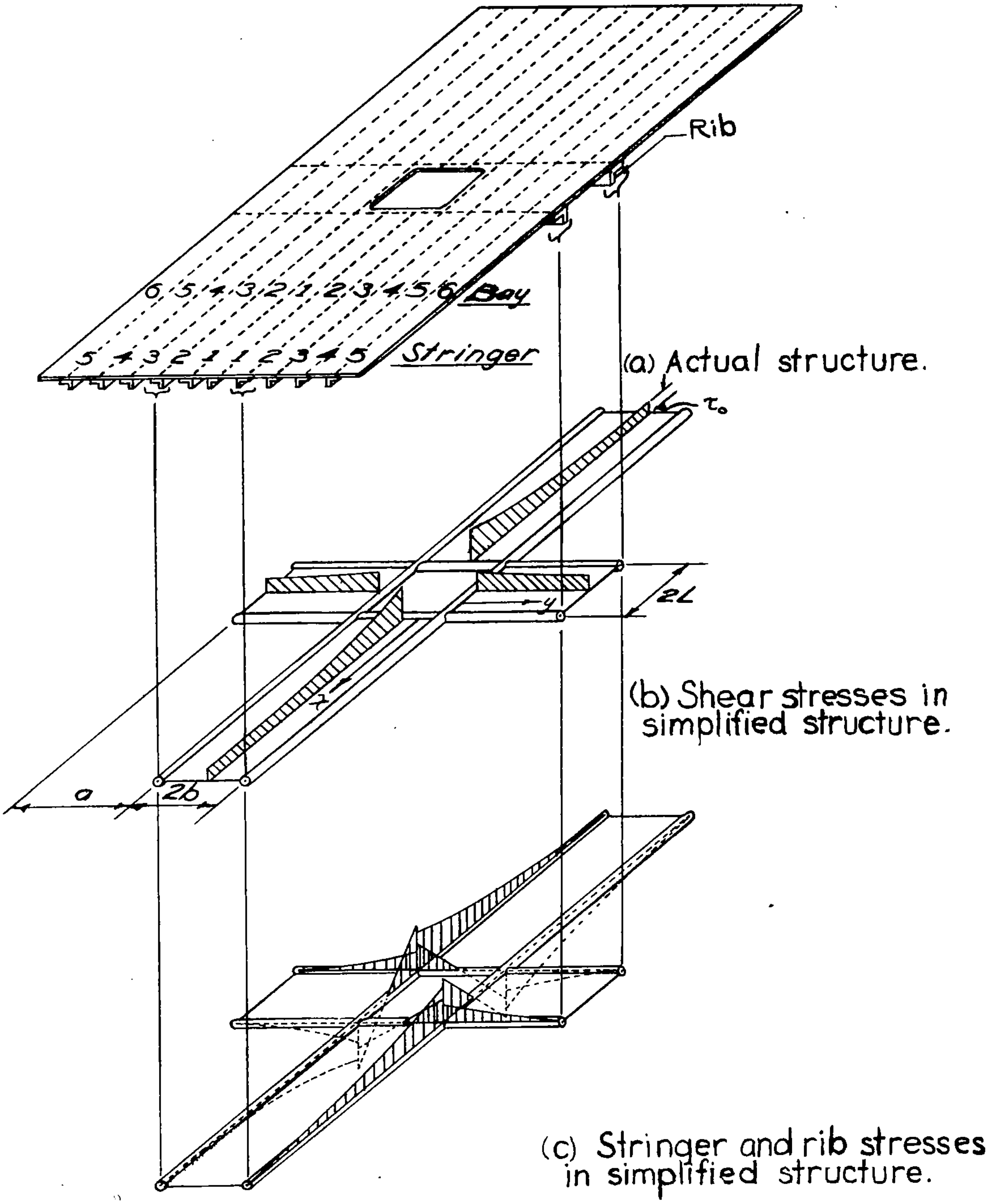

Figure 2.- Schematic representation of actual structure and of simplified structure with stress distributions.

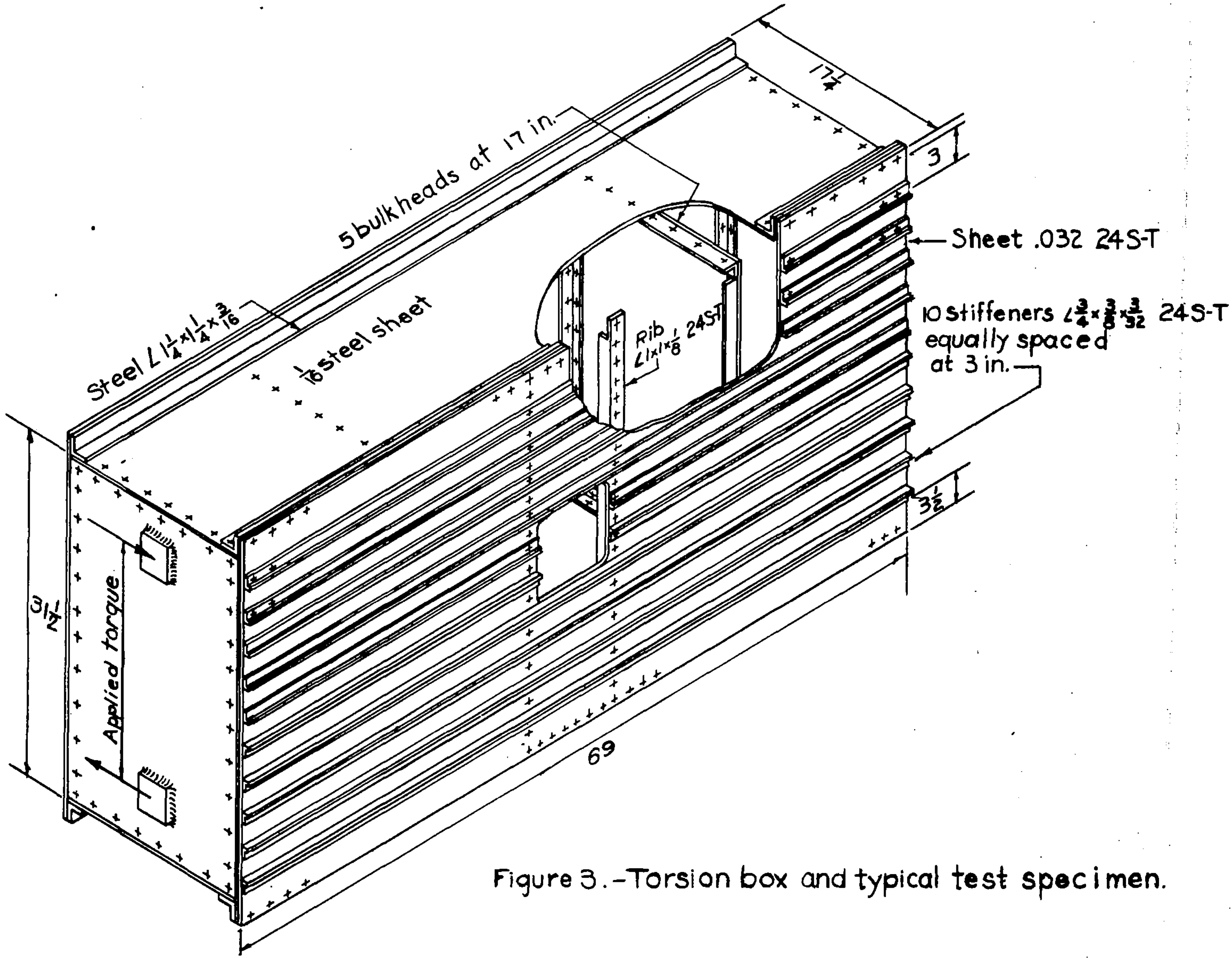

Figure 3.-Torsion box and typical test specimen.

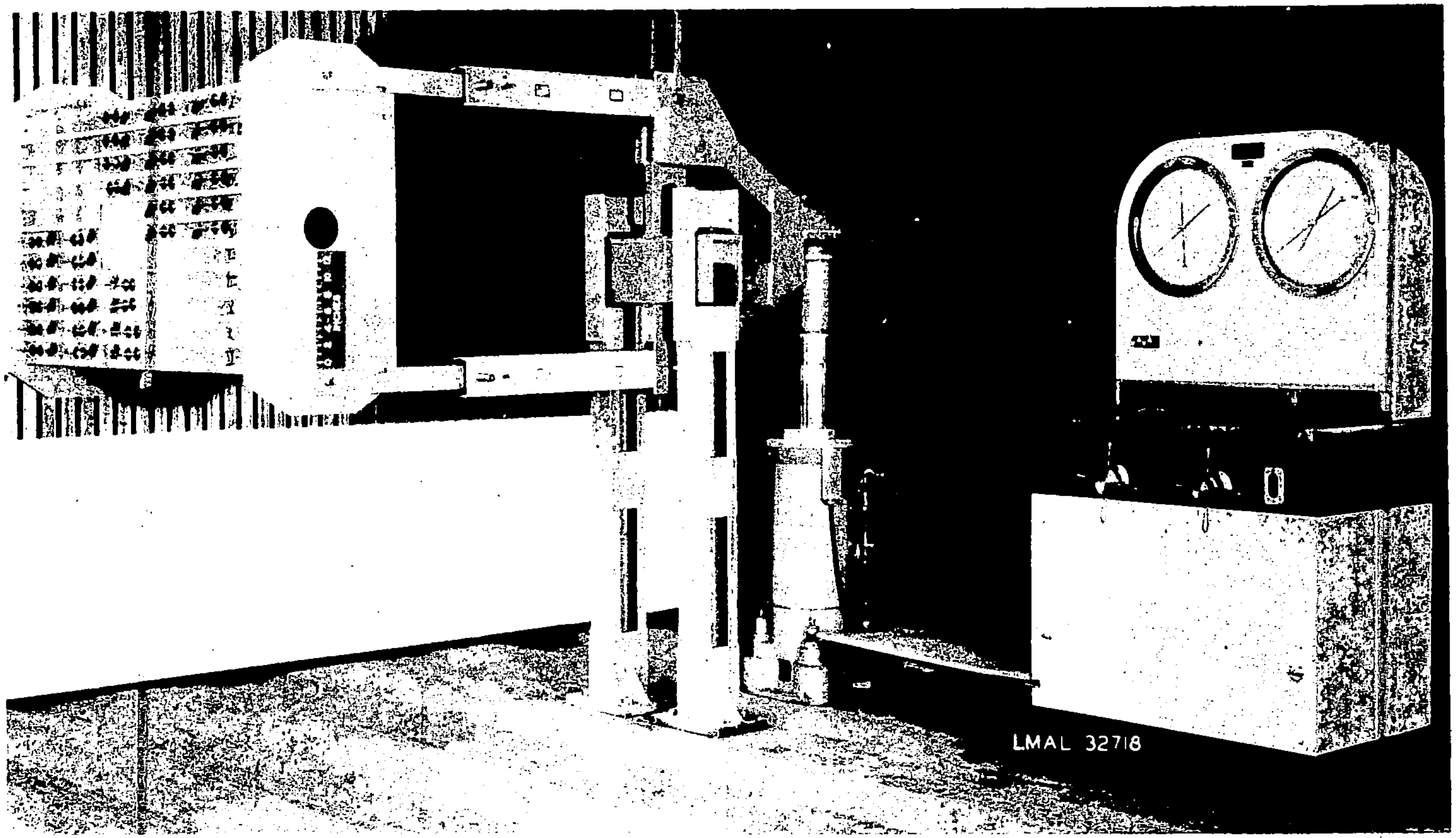

Figure 4.- Torsion box under load.

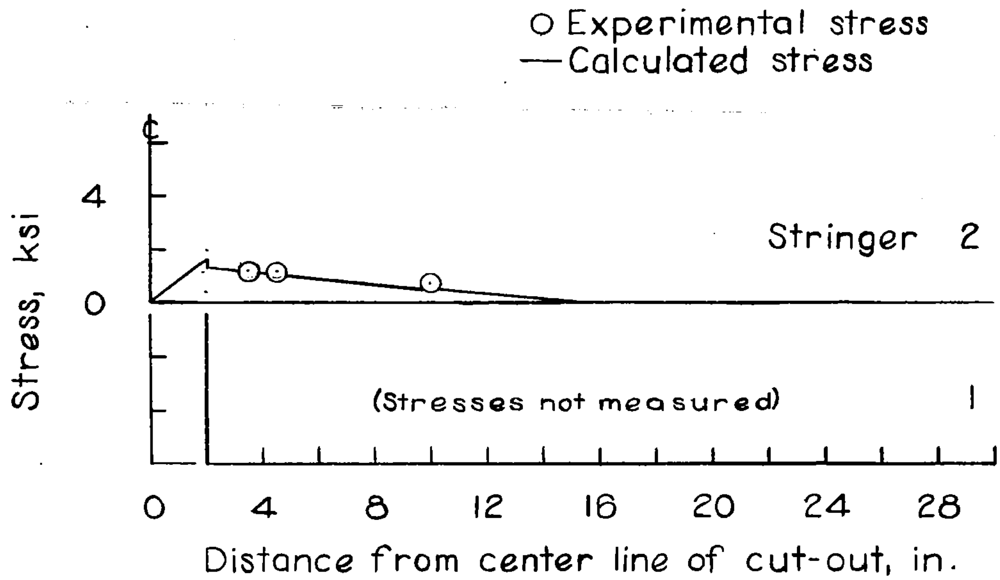

Figure 5.- Stringer stresses in panel with cut-out when L=2 and b=4½, test 2.

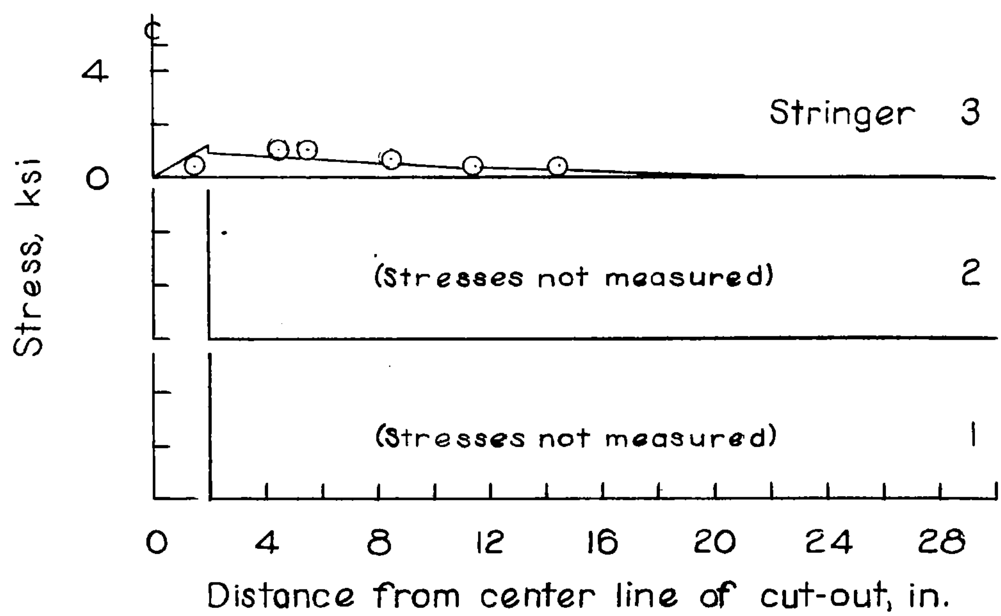

Figure 6.- Stringer stresses in panel with cut-out when L=2 and b=7½, test 3.

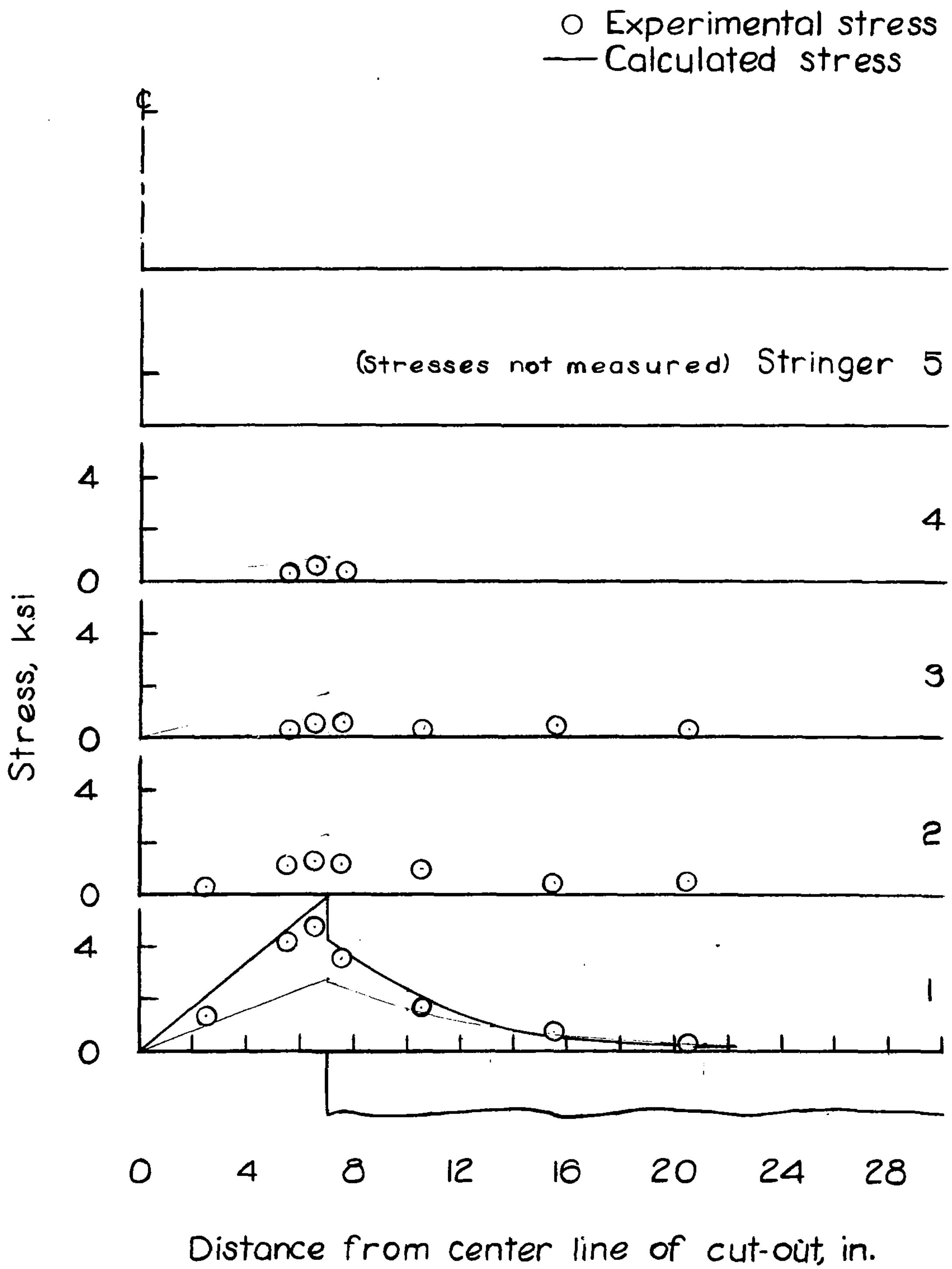

Figure 7.- Stringer stresses in panel with cut-out when L=7 and b=1½, test 4.

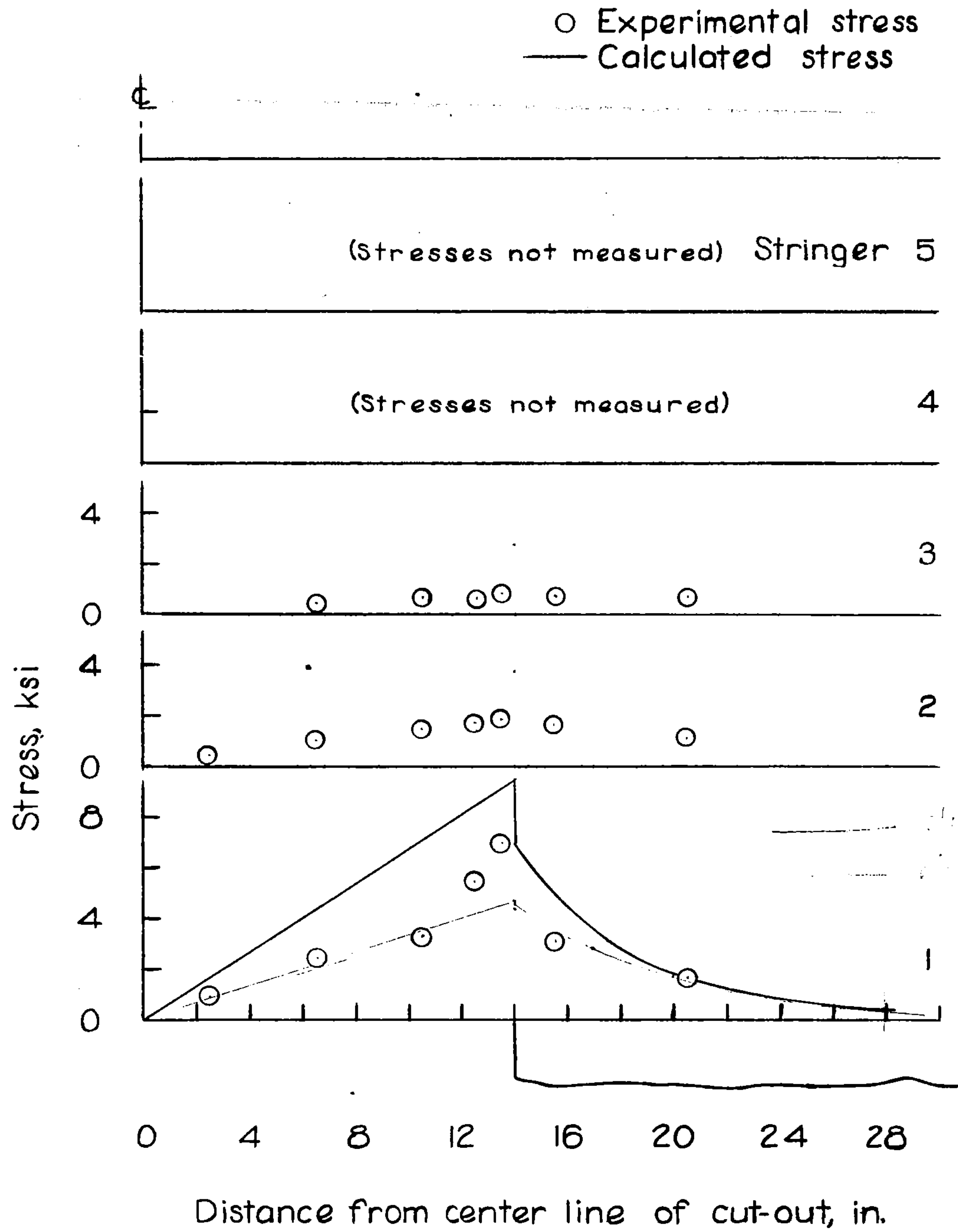

Figure 8.- Stringer stresses in panel with cut-out when L= 14 and b= $1\frac{1}{2}$, test 5. (Theoretical curve of doubtful accuracy because L→ w.)

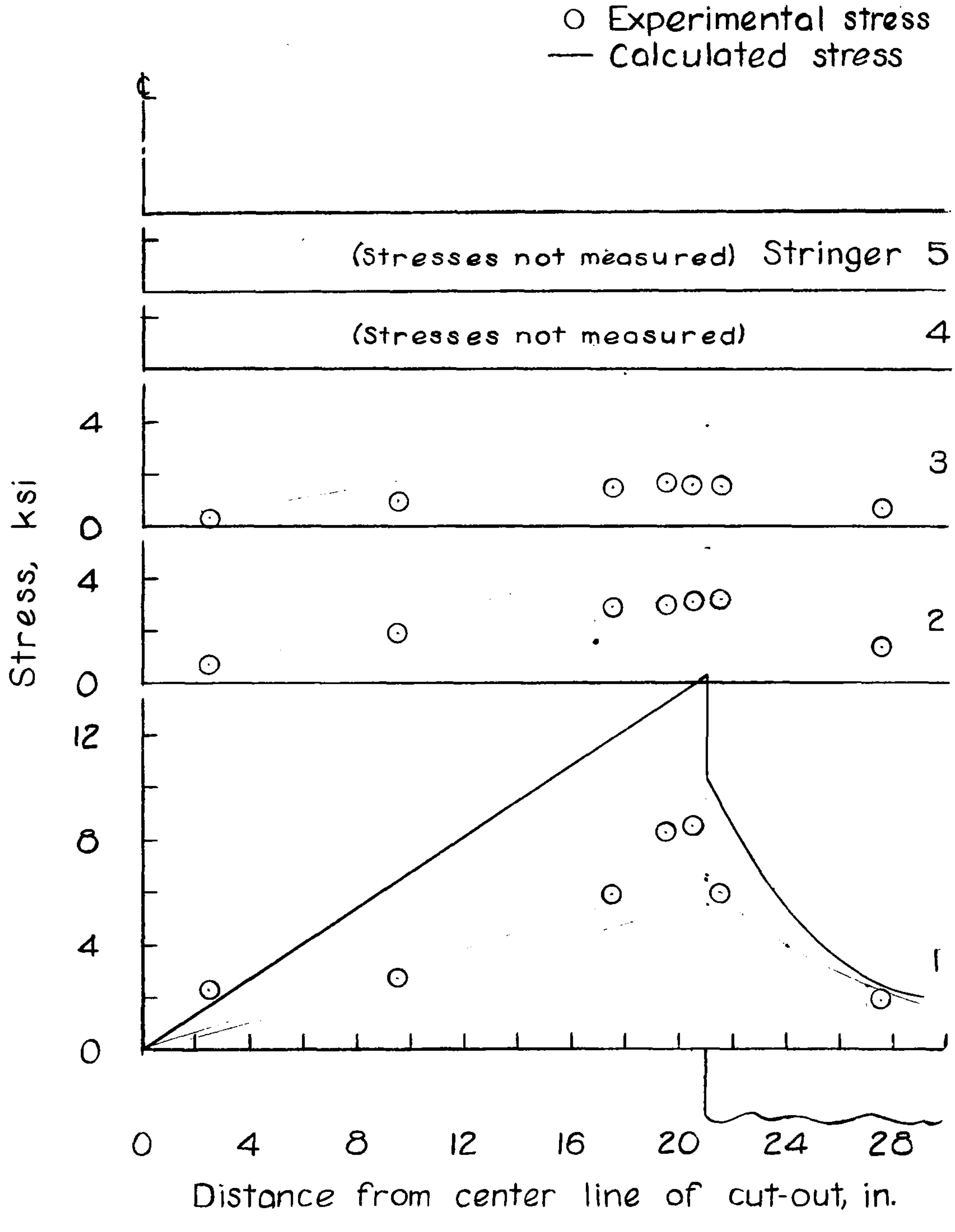

Figure 9.- Stringer stresses in panel with cut-out when L = 21 and b = 1½, test 6. (Theoretical curve inapplicable because L > w.)

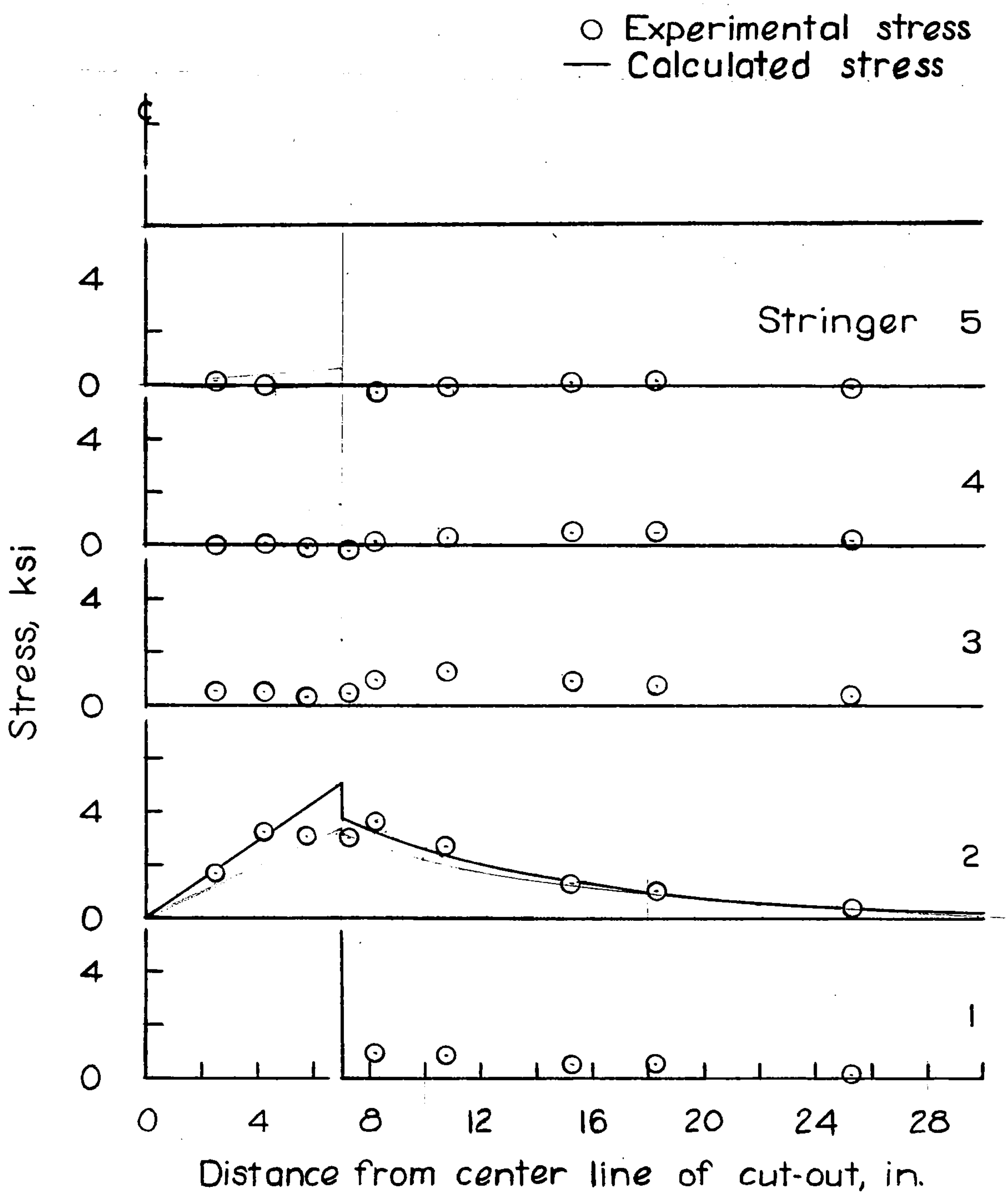

Figure 10.- Stringer stresses in panel with cut-out when L=7 and b=4½, test 7.

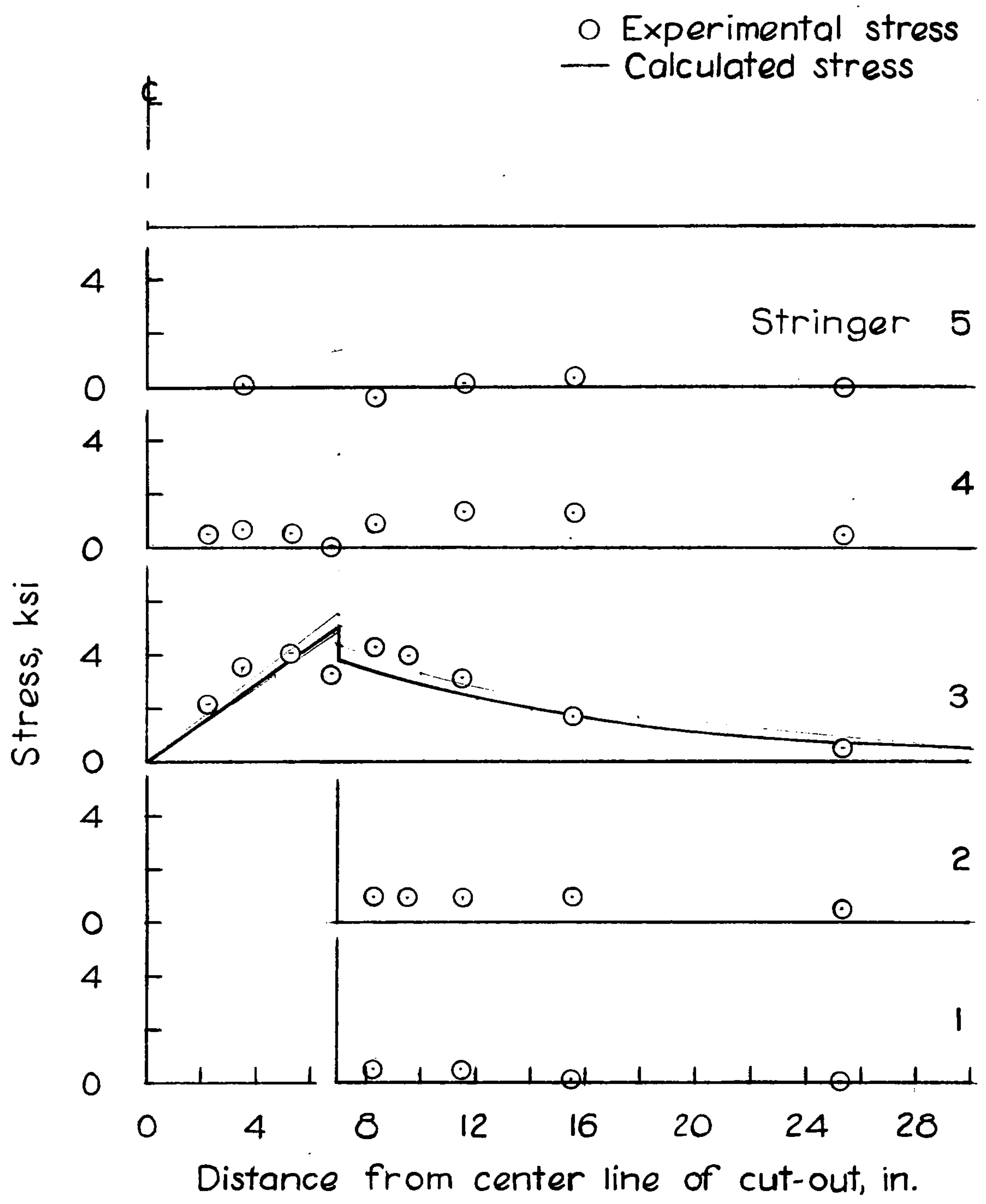

Figure 11.- Stringer stresses in panel with cut-out when L = 7 and b = 7½, test 8.

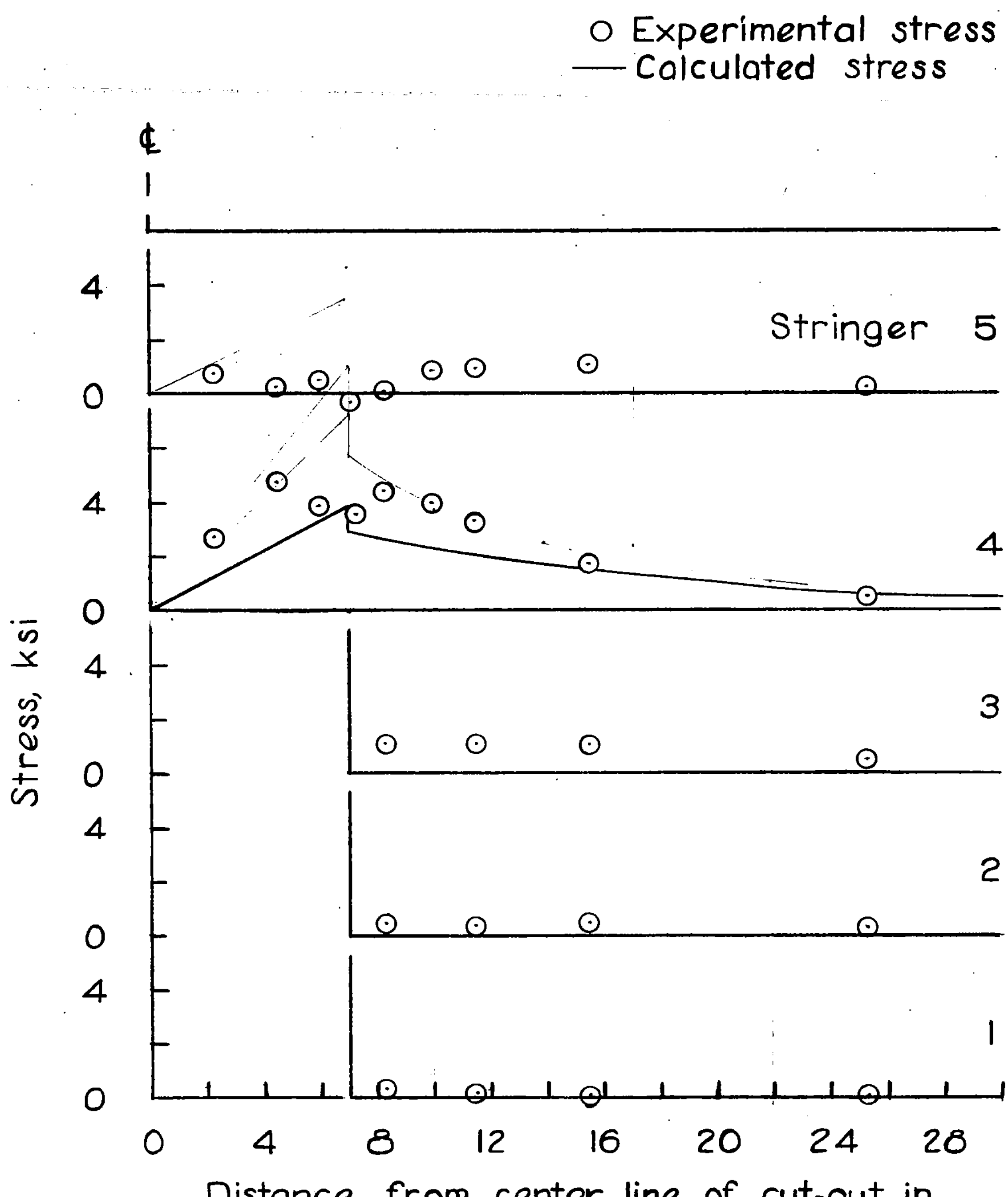

Figure 12.- Stringer stresses in panel with cut-out when L=7 and $b = 10\frac{1}{2}$, test 9. (Theoretical curve inapplicable because 2b> w.)

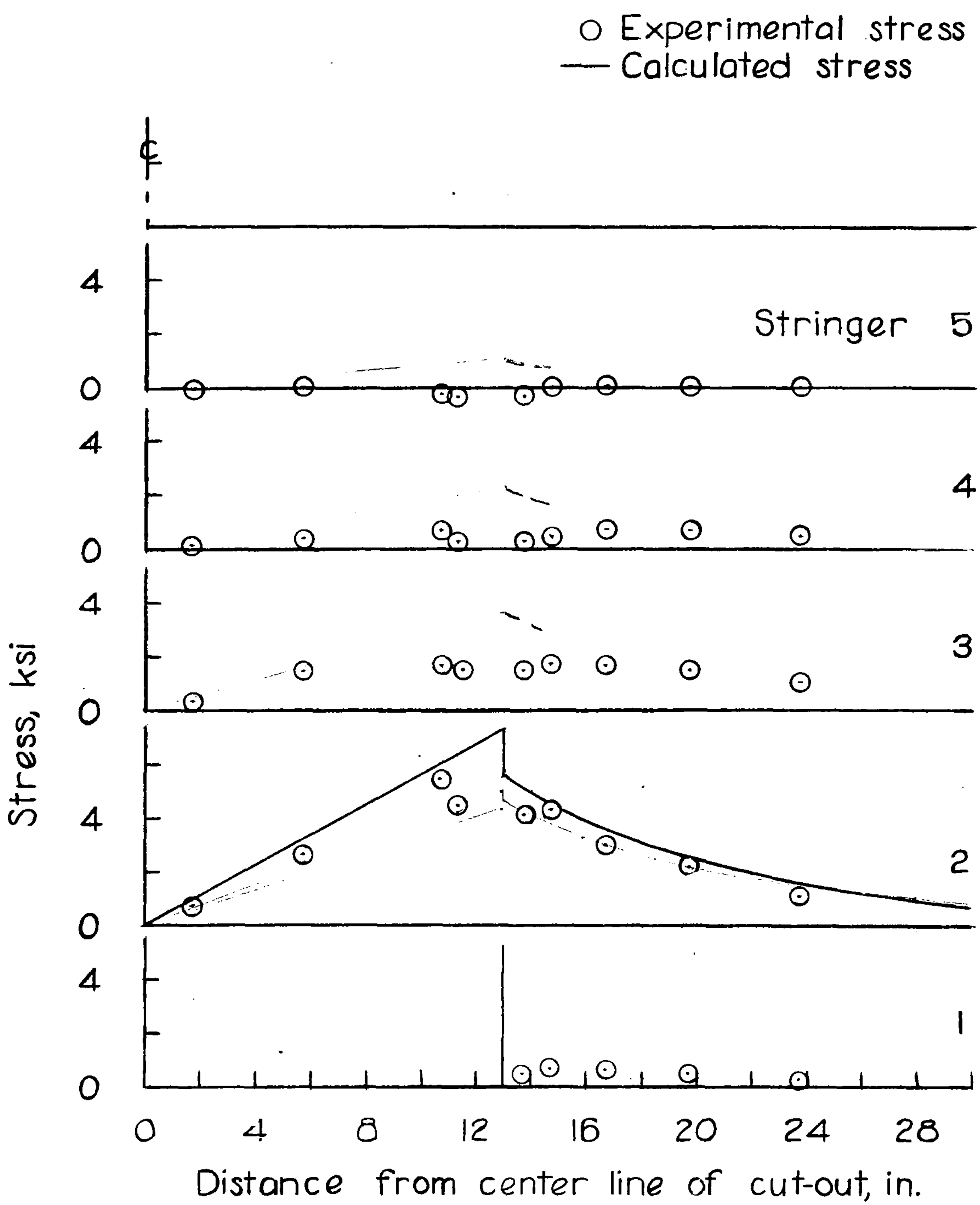

Figure 13.- Stringer stresses in panel with cut-out when L=13 and b=4½, test 10.

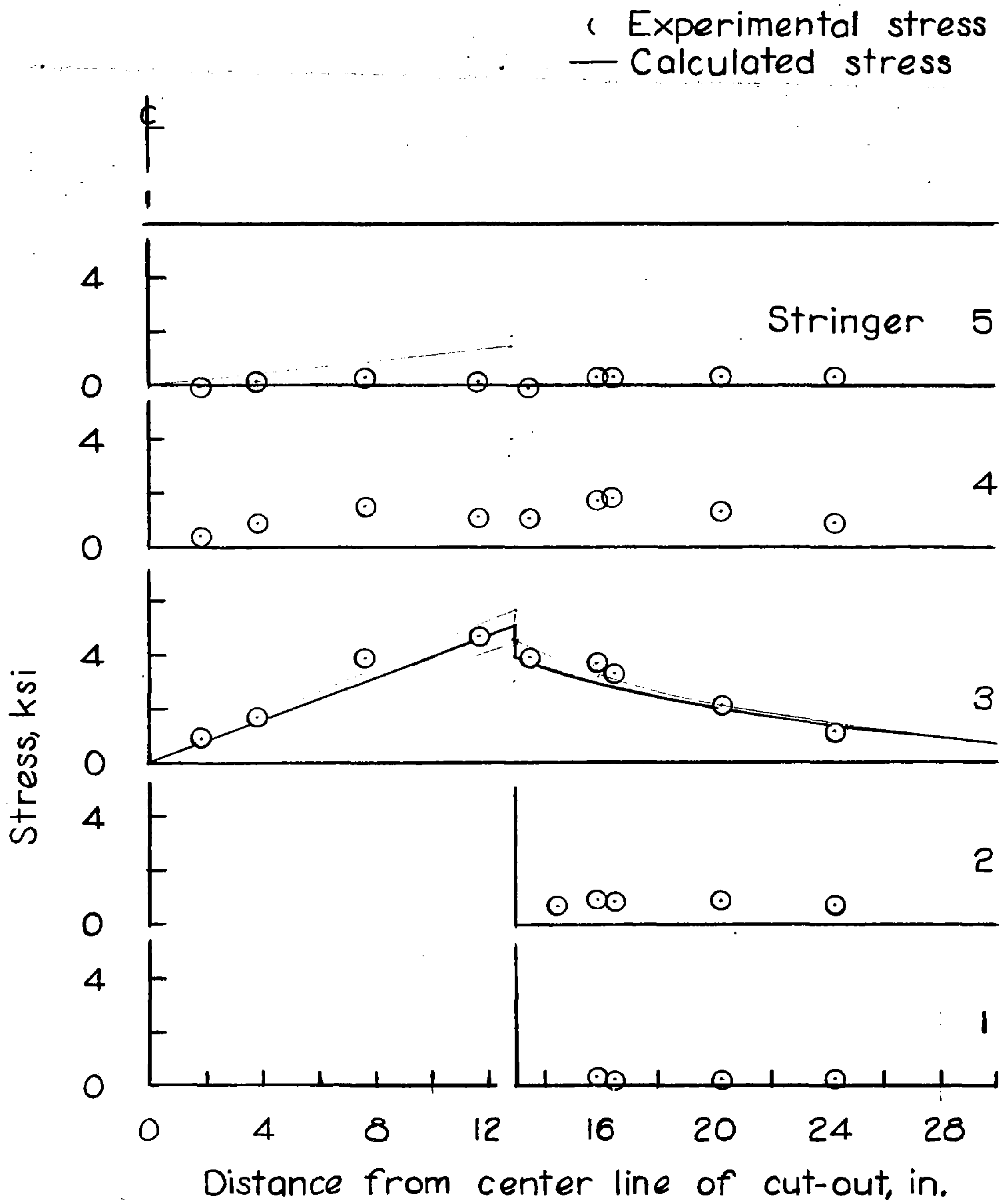

Figure 14.- Stringer stresses in panel with cut-out when L = 13 and b = $7\frac{1}{2}$, test 11.

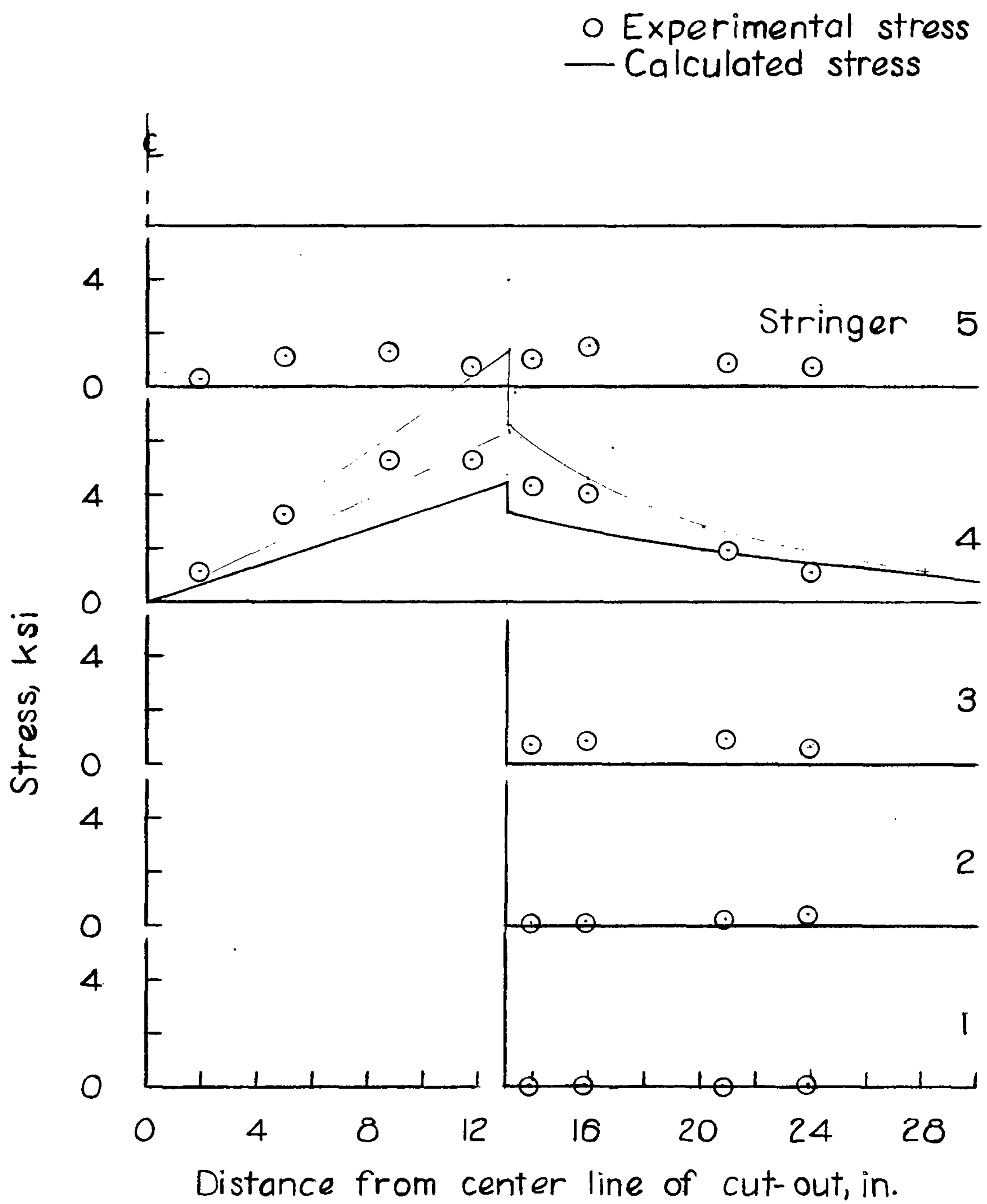

Figure 15.- Stringer stresses in panel with cut-out when L= 13 and b= $10\frac{1}{2}$, test 12. (Theoretical curve inapplicable because 2b>w.)

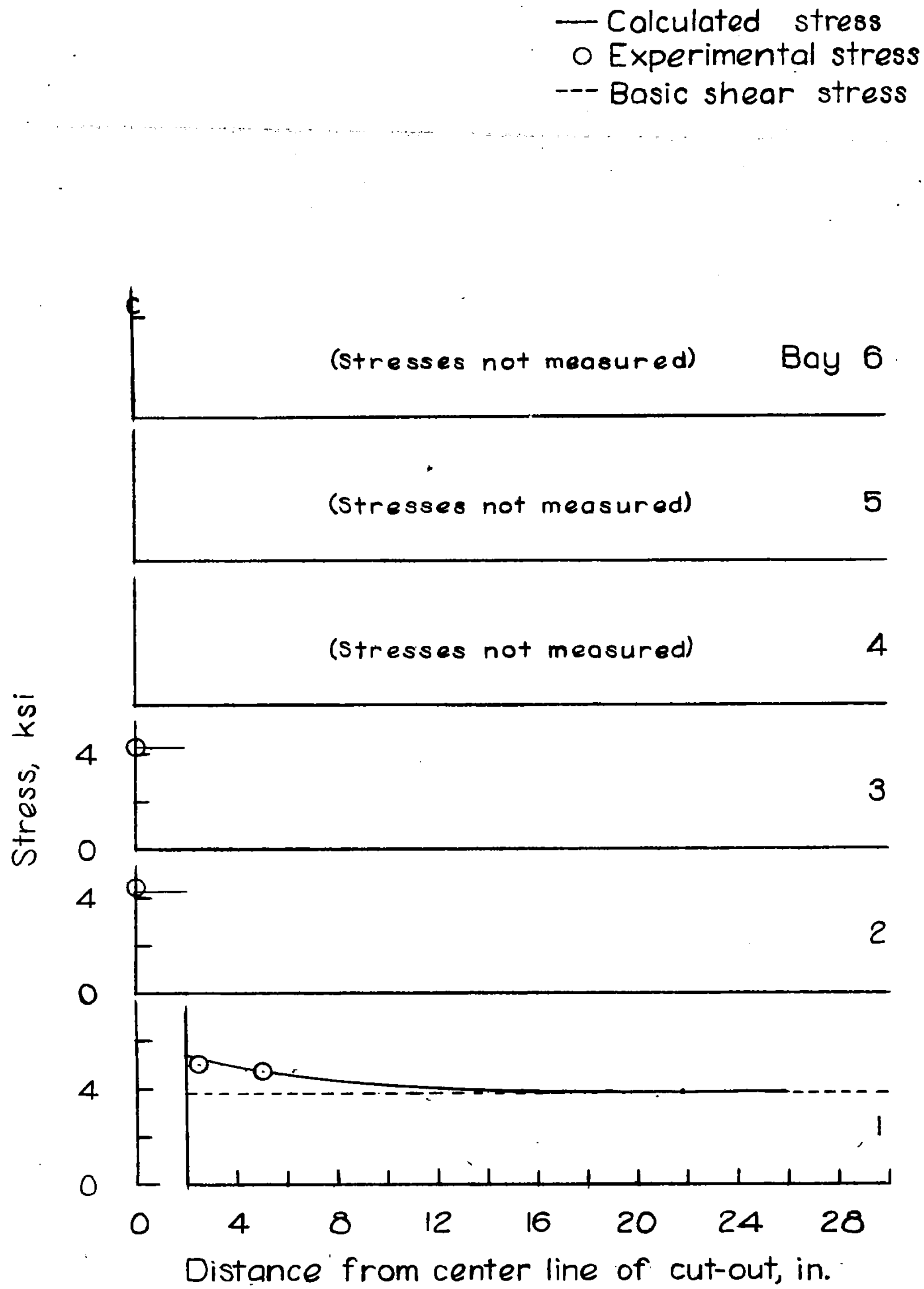

Figure 16.- Shear stresses in panel with cut-out when L=2 and b=1½, test 1.

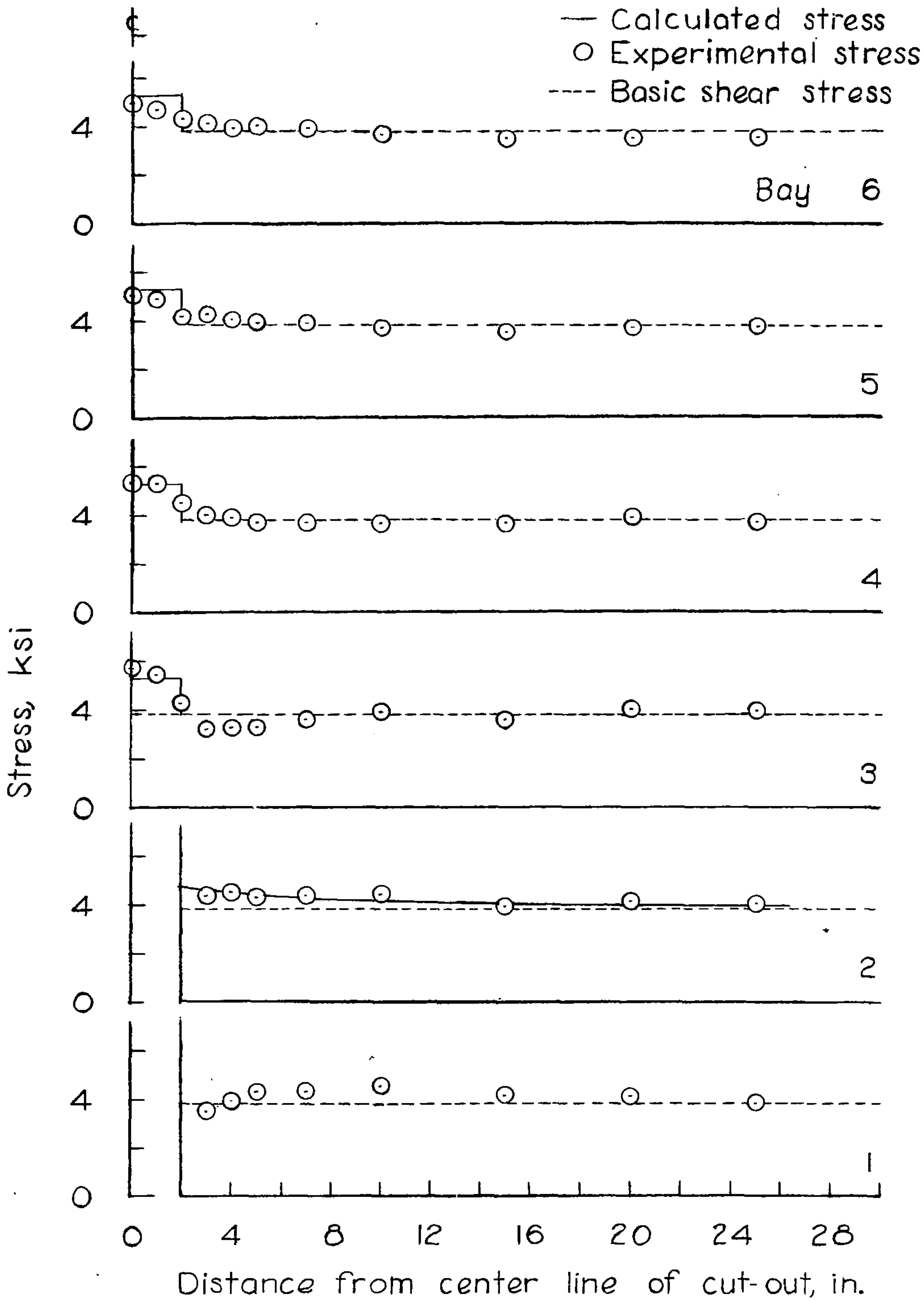

Figure 17.- Shear stresses in panel with cut-out when L=2 and b=4½, test 2.

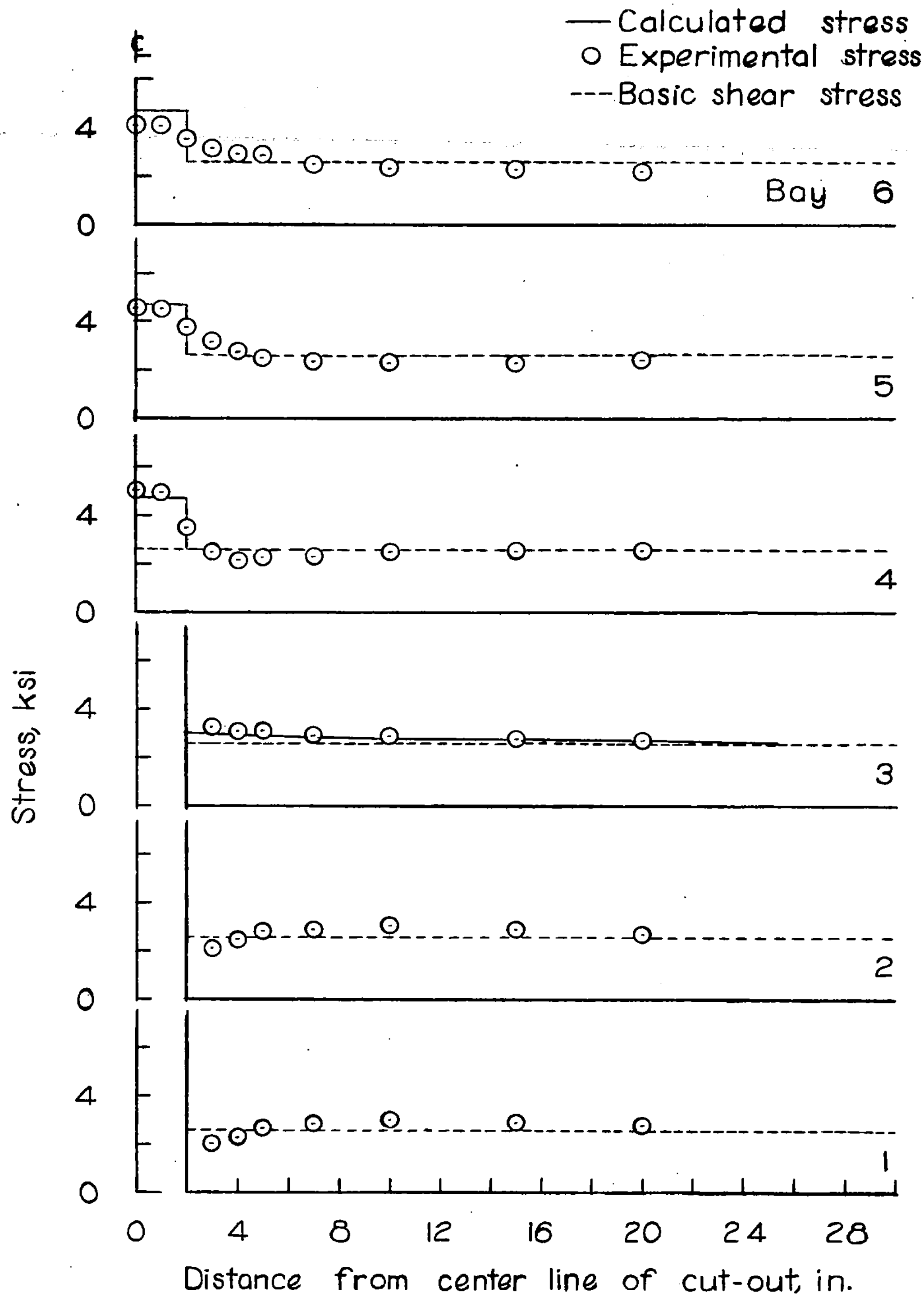

Figure 18.- Shear stresses in panel with cut-out when L=2 and b=7½, test 3.

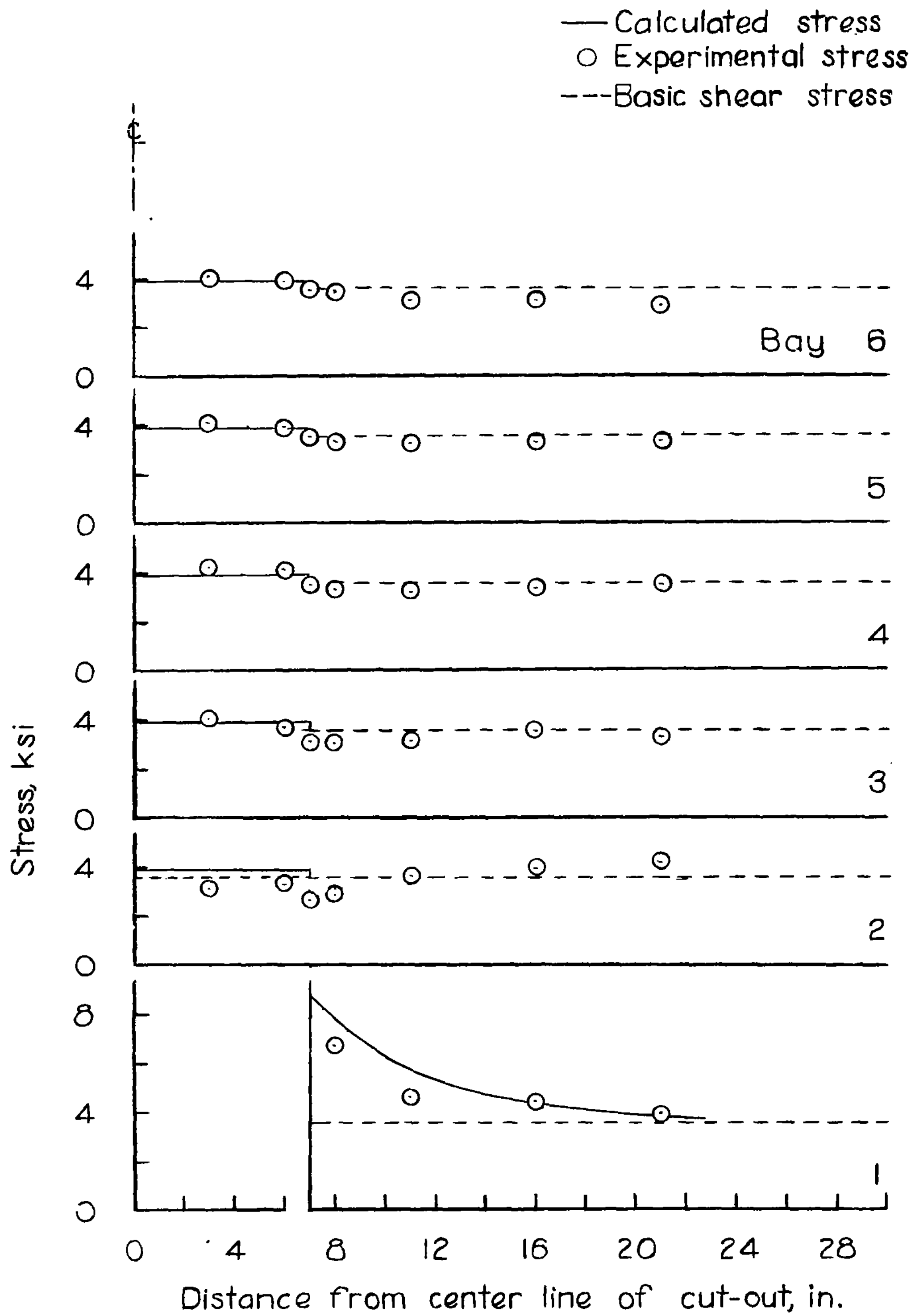

Figure 19.- Shear stresses in panel with cut-out when L = 7 and b = 1½, test 4.

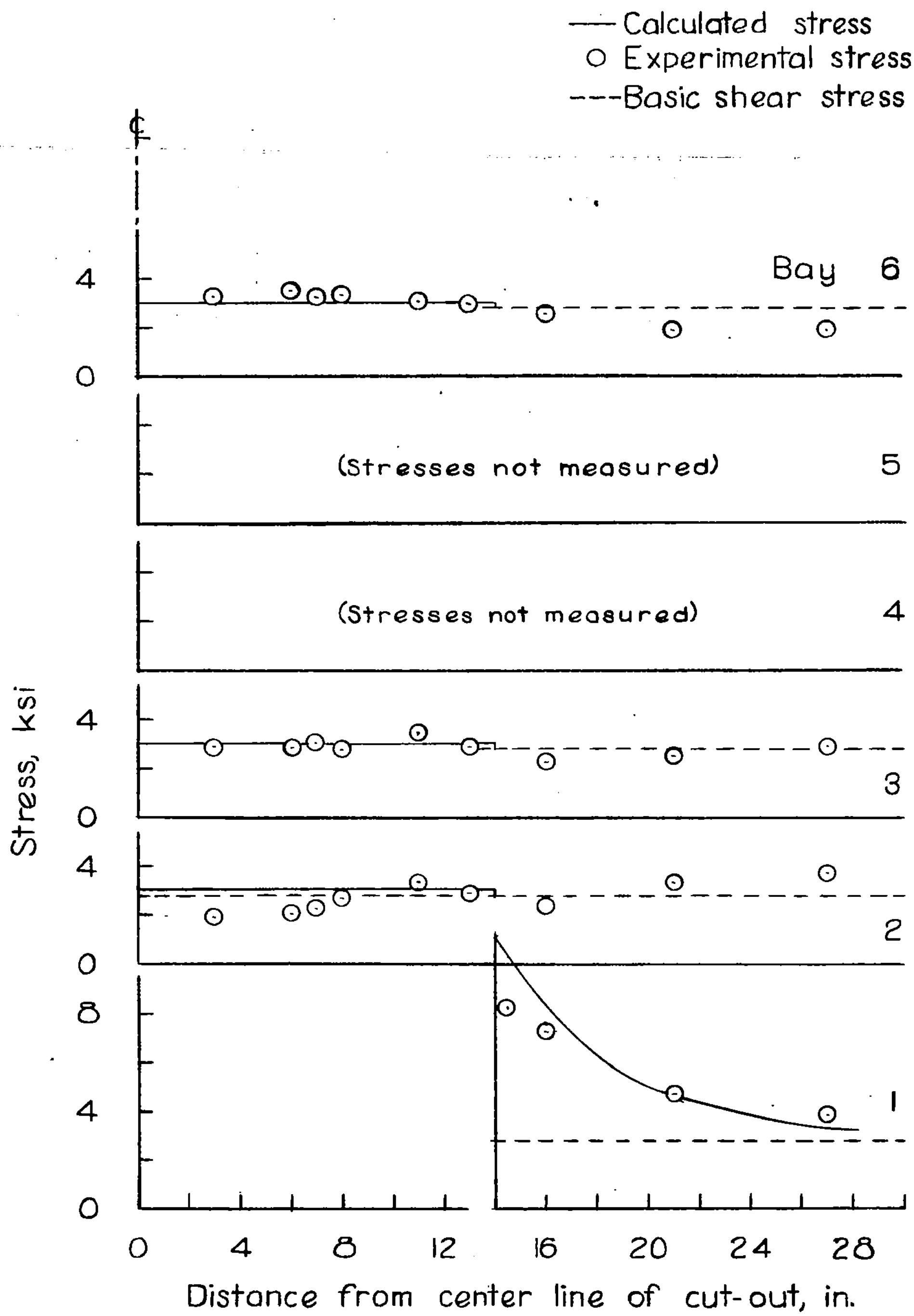

Figure 20.- Shear stresses in panel with cut-out when L=14 and b=1½, test 5. (Theoretical curve of doubtful accuracy because L→w.)

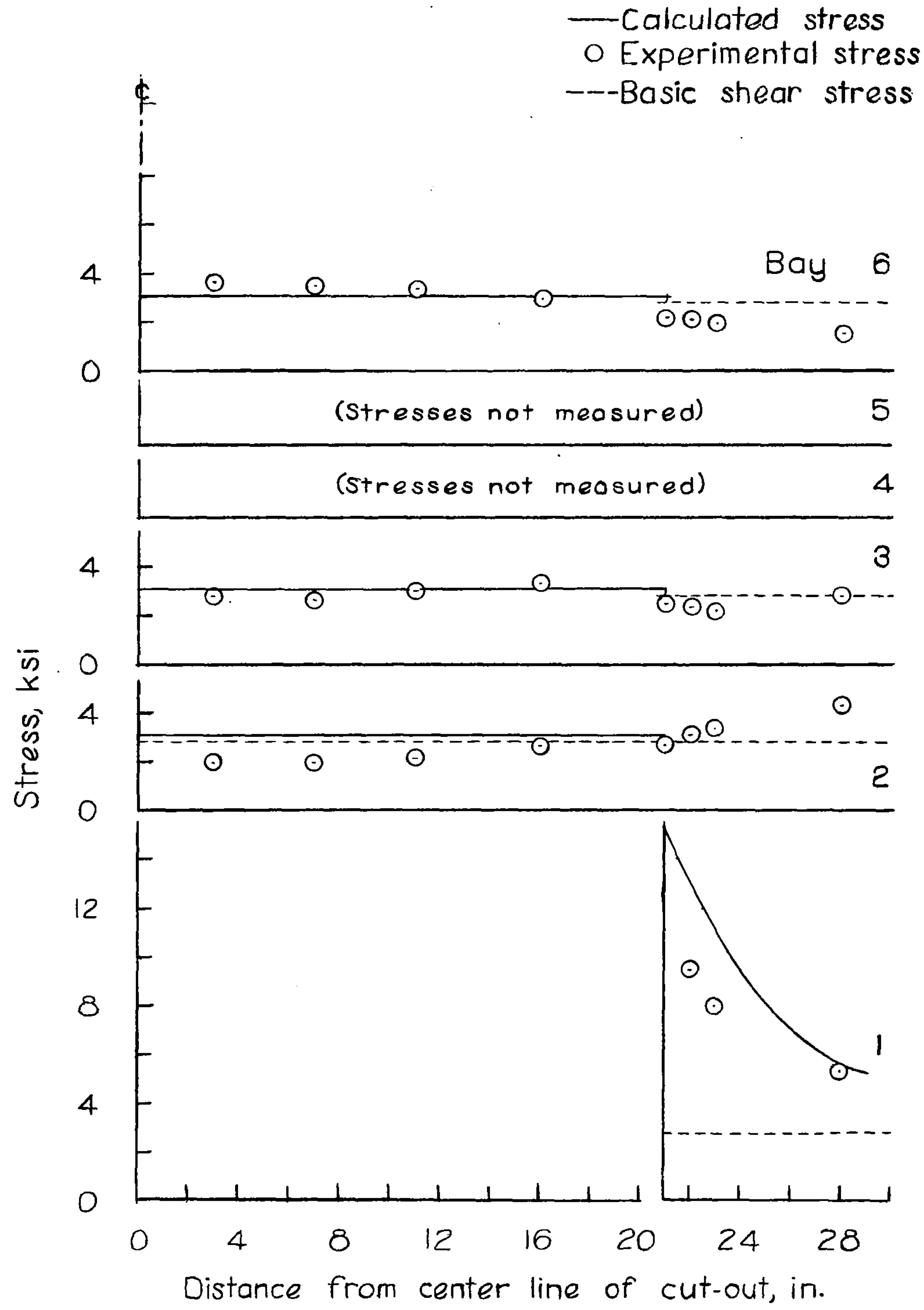

Figure 21.- Shear stresses in panel with cut-out when L=21 and b=$1\frac{1}{2}$, test 6. (Theoretical curve inapplicable because L > w.)

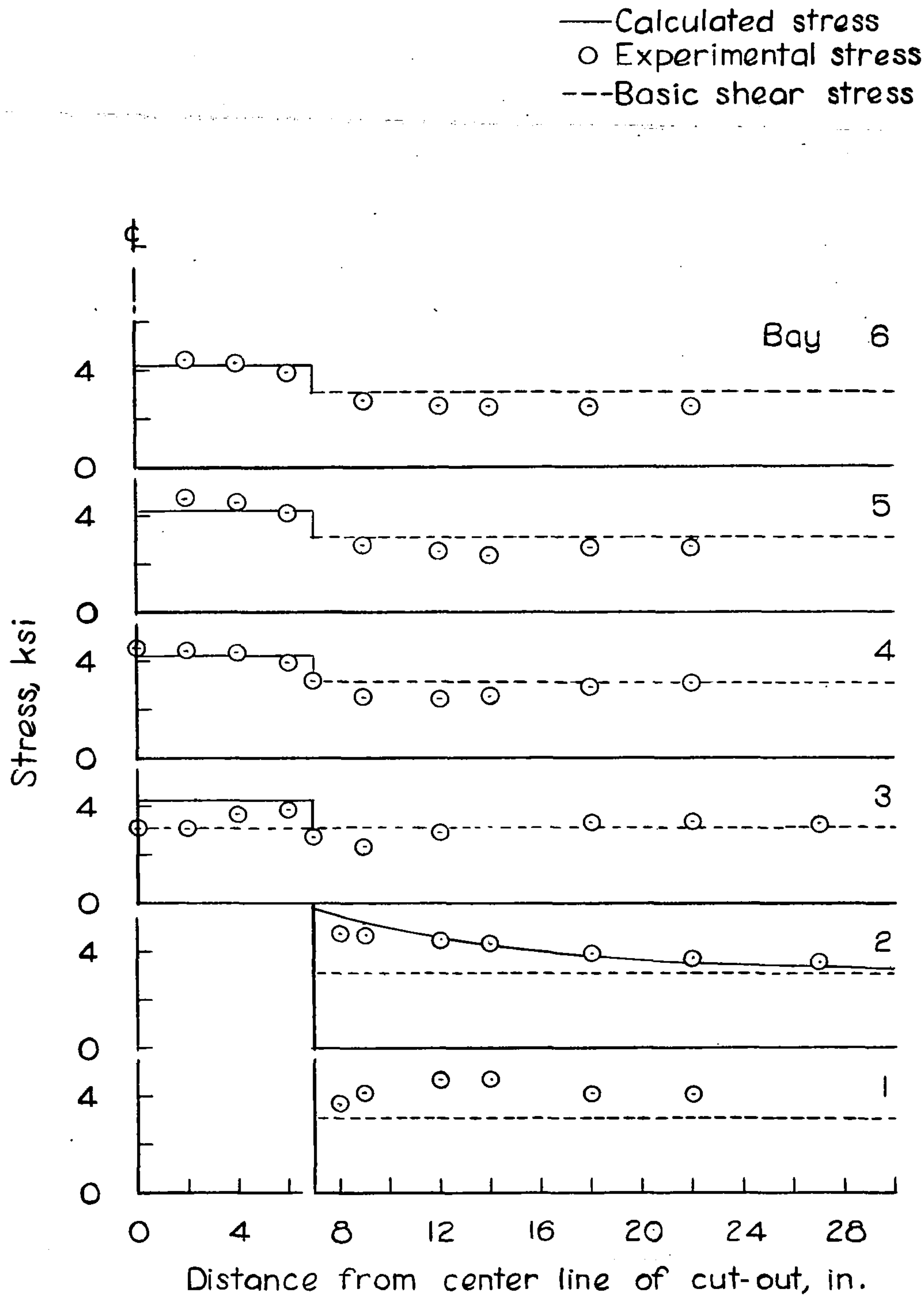

Figure 22.- Shear stresses in panel with cut-out when L = 7 and b = 4½, test 7.

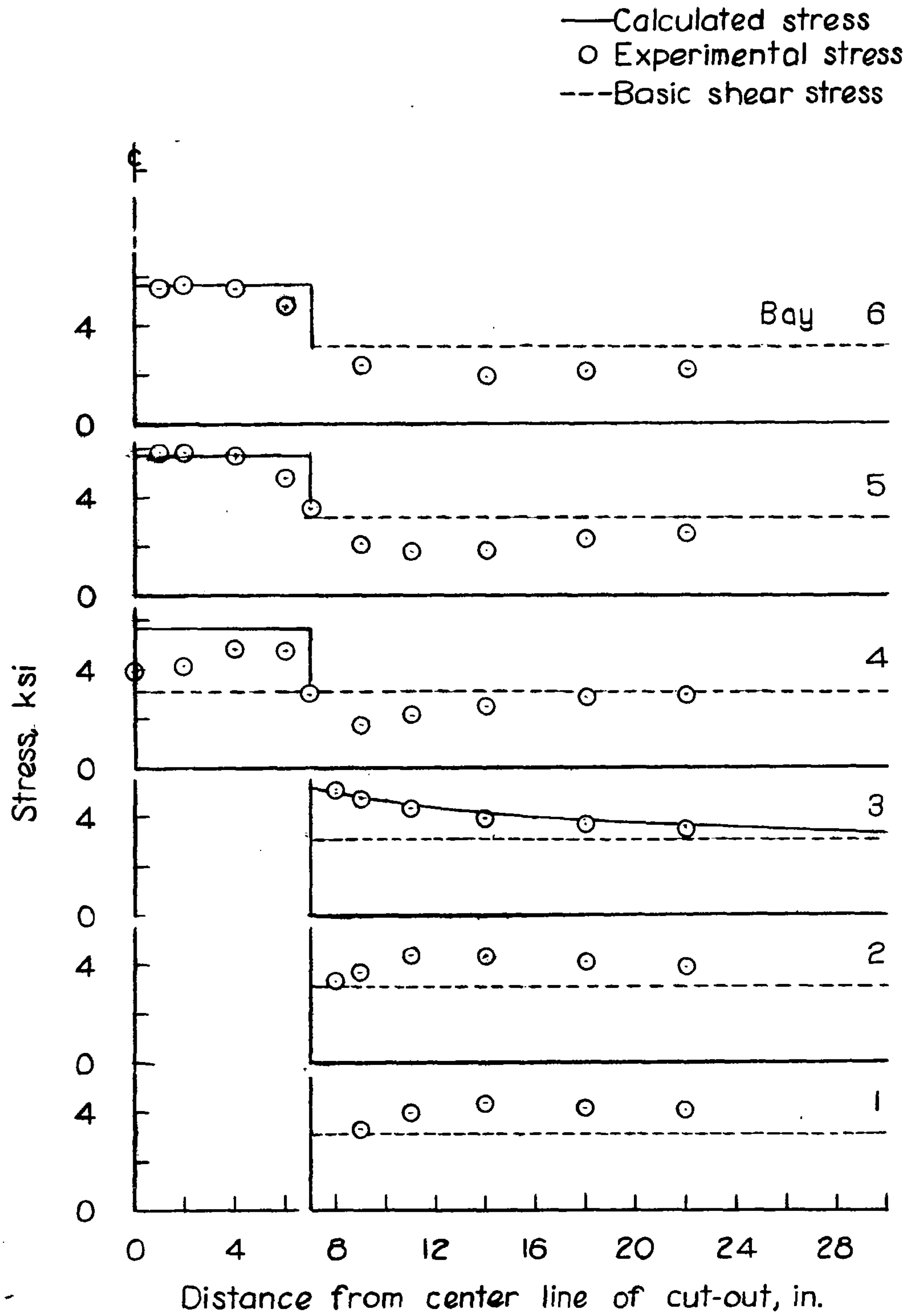

Figure 23.- Shear stresses in panel with cut-out when L=7 and b=7½, test 8.

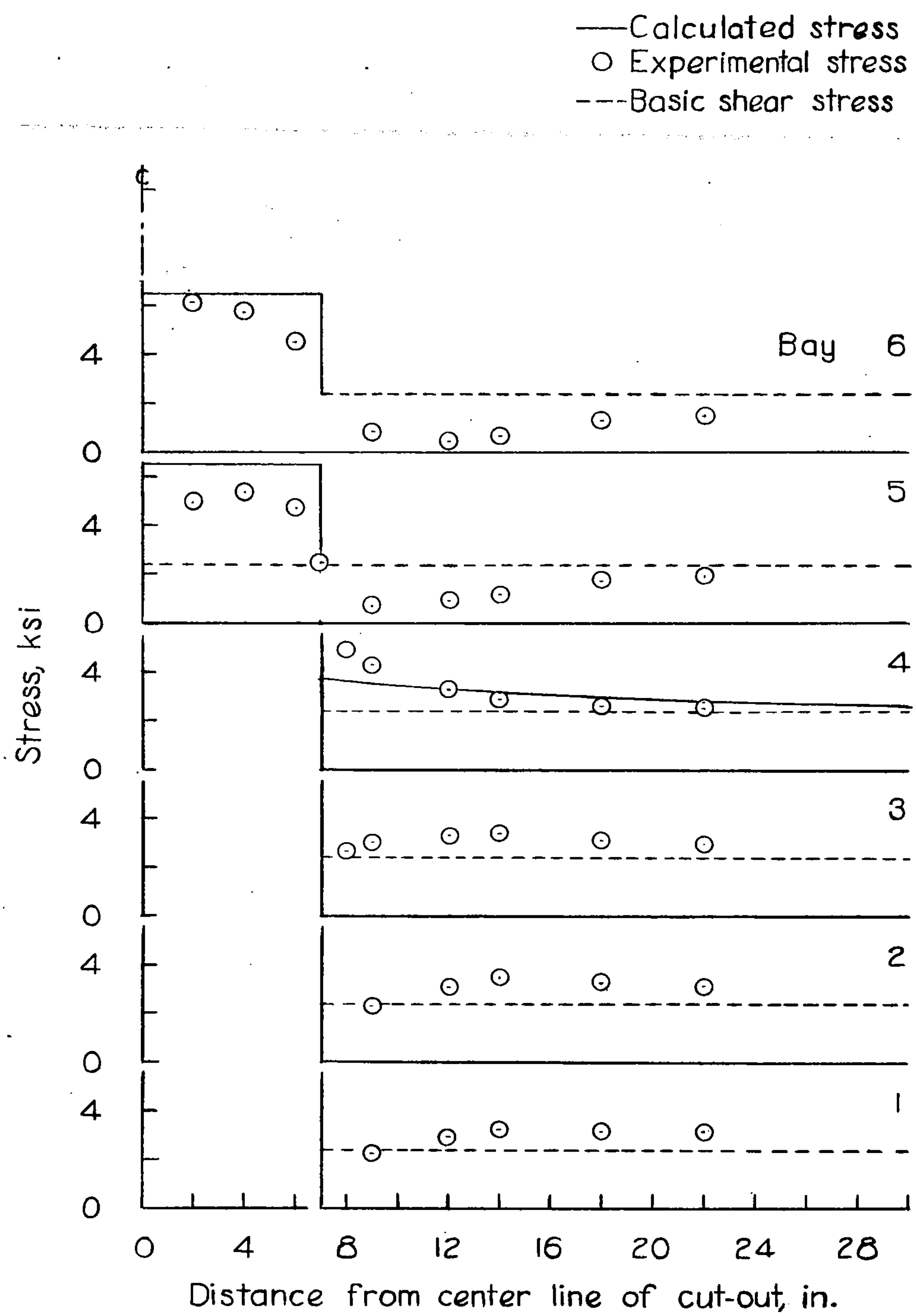

Figure 24.- Shear stresses in panel with cut-out when L=7 and b=10½, test 9. (Theoretical curve inapplicable because 2b>w.)

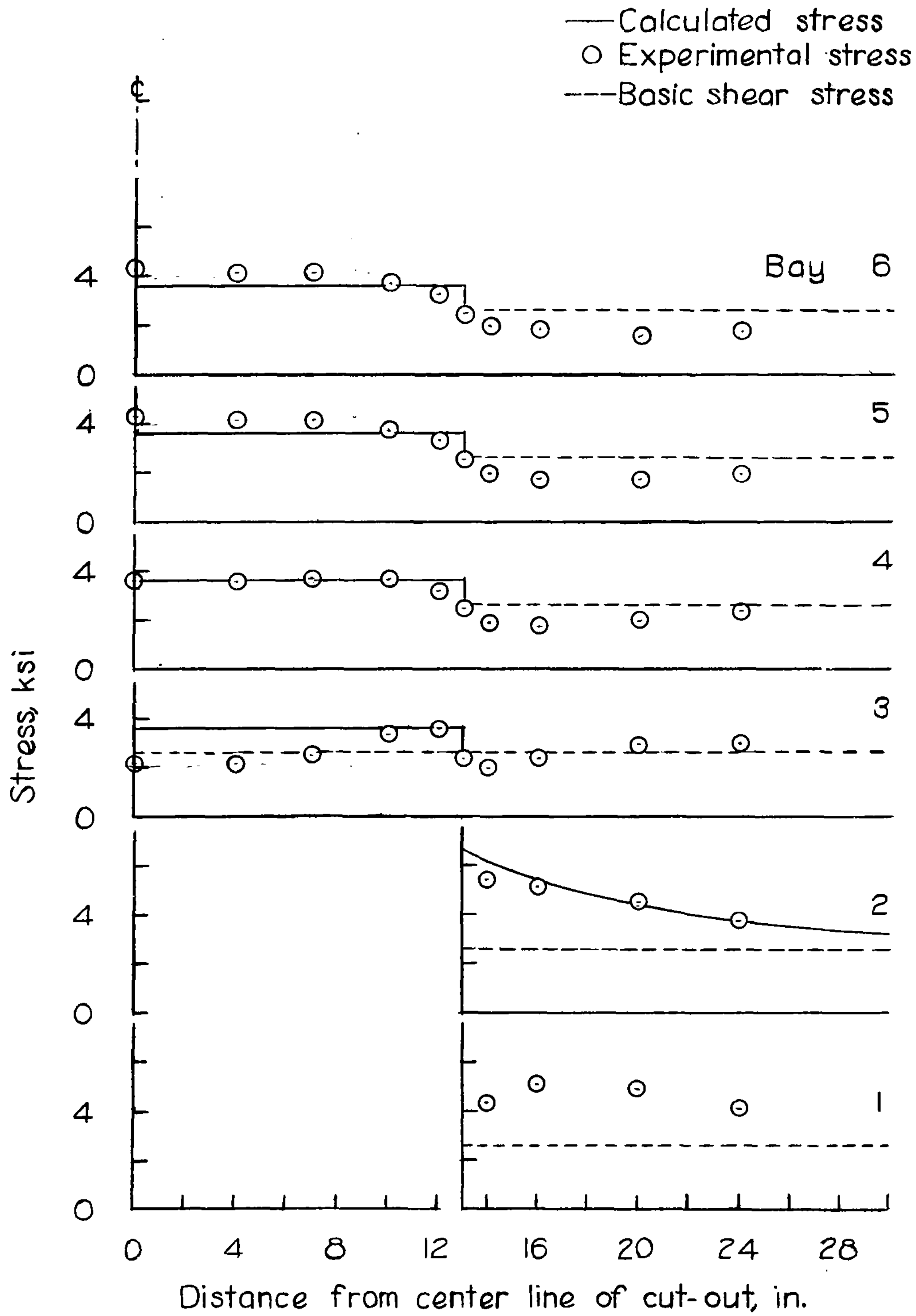

Figure 25.- Shear stresses in panel with cut-out when L = 13 and b = 4½, test 10.

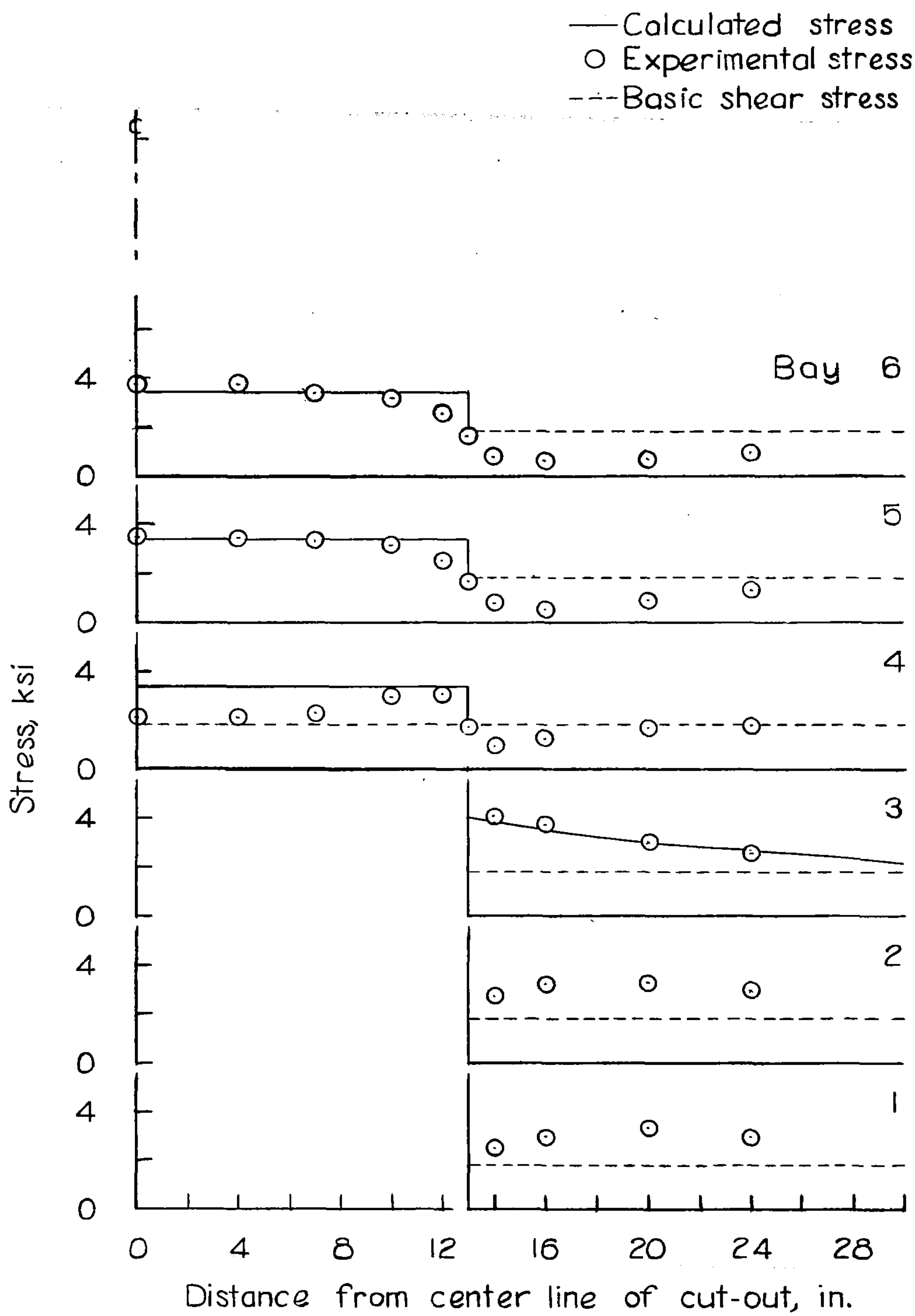

Figure 26.- Shear stresses in panel with cut-out when L=13 and b=7 $\frac{1}{2}$, test 11.

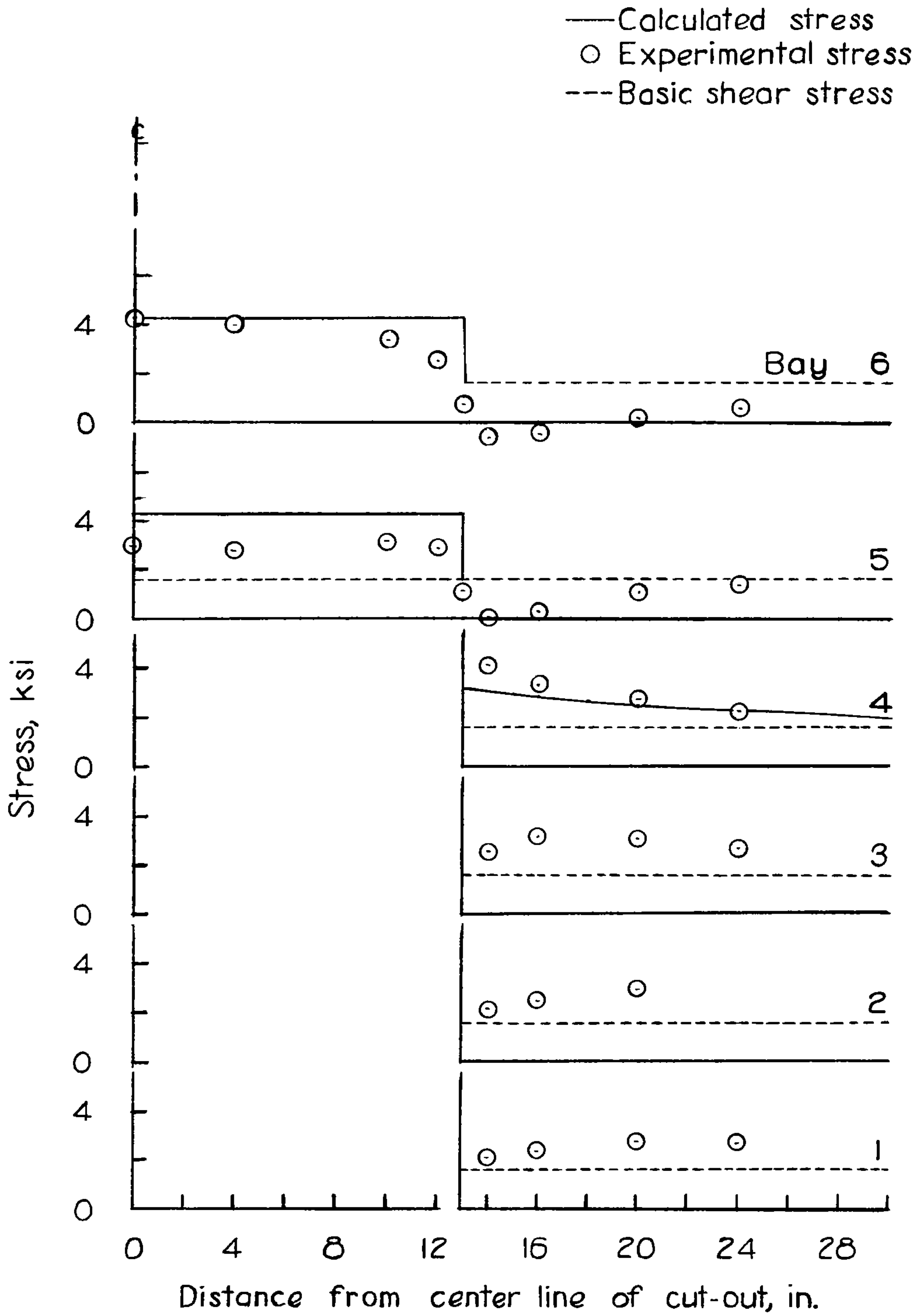

Figure 27.- Shear stresses in panel with cut-out when L = 13 and b = 10½, test 12. (Theoretical curve inapplicable because 2b>w.)